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Stationary BPS solutions in $N = 2$ supergravity with R^2-interactions

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Abstract: We analyze a broad class of stationary solutions with residual $N = 1$ supersymmetry of four-dimensional $N = 2$ supergravity theories with terms quadratic in the Weyl tensor. These terms are encoded in a holomorphic function, which determines the most relevant part of the action and which plays a central role in our analysis. The solutions include extremal black holes and rotating field configurations, and may have multiple centers. We prove that they are expressed in terms of harmonic functions associated with the electric and magnetic charges carried by the solutions by a proper generalization of the so-called stabilization equations. Electric/magnetic duality is manifest throughout the analysis. We also prove that spacetimes with unbroken supersymmetry are fully determined by electric and magnetic charges. This result establishes the so-called fixed-point behavior according to which the moduli fields must flow towards certain prescribed values on a fully supersymmetric horizon, but now in a more general context with higher-order curvature interactions. We briefly comment on the implications of our results for the metric on the moduli space of extremal black hole solutions.

Keywords: Black Holes, Extended Supersymmetry.

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Contents

1. Introduction

In this paper we determine a broad class of stationary solutions of four-dimensional $N = 2$ supergravity theories with R^2-interactions. The supergravity theories that we consider are based on vector multiplets and hypermultiplets coupled to the supergravity fields and contain the standard Einstein-Hilbert action as well as terms quadratic in the Weyl tensor. The most relevant part of the interactions is encoded in a holomorphic function, which plays a central role in our analysis. The solutions that we consider are BPS solutions, because they possess a residual $N = 1$ supersymmetry. Some of them describe extremal black holes that carry electric and/or magnetic charges or superpositions thereof. We also describe rotating solutions with one or several centers. The extremal black holes are solitonic interpolations between two fully supersymmetric groundstates. Without R^2-interactions these are flat Minkowski spacetime at spatial infinity and a Bertotti-Robinson geometry at the horizon. In that case, the moduli fields, which can take arbitrary values at infinity, must flow to specific values at the horizon which are determined in terms of the charges. This so-called fixed-point behavior explains why the black hole entropy depends only on the charges and not on the asymptotic values of the moduli. This is in contradistinction with the black hole mass which does depend on the values of the fields at spatial infinity. Owing to this fixed-point behavior the resulting expressions for the entropy, based on the effective low-energy action, can be compared successfully with microstate counting results from string and brane theory, which also depend exclusively on the charges.

Solutions based on supergravity actions without R^2-terms were analyzed some time ago [1]–[4]. Important features of solutions with R^2-interactions were presented more recently in a number of papers [5, 6, 7] and reviewed in [8]. In [6] we showed that corrections to the black hole entropy associated with R^2-terms are in agreement with certain subleading corrections to the entropy (in the limit of large charges) that follow from the counting of microstates [9]. The main ingredients of the derivation in [6] are the behavior of the solution at the horizon and the use of a definition of the black hole entropy that is appropriate when R^2-interactions are present. In order to ensure the validity of the first law of black hole mechanics, we used the definition provided by the Noether method of [10]. This definition leads to an entropy that deviates from the Bekenstein-Hawking area law.

The purpose of this work is to present the complete proof underlying the results of [6] and to further extend our study of solutions in the presence of R^2-interactions. In particular we consider the full interpolating extremal black hole solution, multi-centered solutions, as well as general stationary solutions. All solutions known so far (e.g. [1, 2, 3, 11]) are contained as special cases. We begin our analysis by determining all spacetimes with $N = 2$ supersymmetry. In doing so we systematize and complete the analysis presented in [6]. We prove that, in spite of the presence of R^2-terms, there is still a unique spacetime, which is of the Bertotti-Robinson type, whose radius as well as the values of the various moduli fields are determined by the electric and magnetic charges carried by the solution. Flat Minkowski spacetime can be viewed as a special case of such a solution, but here the moduli are constant and arbitrary and there are no electric and magnetic fields. Our analysis thus shows that the enhancement of supersymmetry at the horizon forces the moduli fields to take prescribed values. Consequently the uniqueness of the horizon geometry implies the existence of a fixed-point behavior even in the presence of R^2-interactions. Note that the fixed-point behavior is usually derived by invoking flow arguments based on the interpolating solutions (see, e.g. [1, 4]), but these arguments are much more difficult to derive in the presence of R^2-interactions.

Subsequently we turn to the analysis of spacetimes with residual $N = 1$ supersymmetry. A general analysis of the conditions for $N = 1$ supersymmetry turns out to be extremely cumbersome. We therefore restrict ourselves to a well-defined class of embeddings of residual supersymmetry and derive the corresponding restrictions

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on the bosonic background configurations. Our analysis is set up in such a way that the presence of the R^2-interactions hardly poses complications. This is so because the R^2-terms are incorporated into the lagrangian by allowing the holomorphic function to depend on an extra holomorphic parameter. Furthermore, by stressing the underlying electric/magnetic duality of the field equations throughout the calculations, the dependence on the R^2-interactions remains almost entirely implicit and does not require much extra attention.

Using the restrictions posed by residual supersymmetry and assuming stationary field configurations we analyze the solutions. We prove that they are expressed in terms of harmonic functions associated with the electric and magnetic charges carried by the solutions, while the spatial dependence of the moduli is governed by so-called "generalized stabilization equations". The latter were first conjectured in [3] and in [5] for the case without and with R^2-interactions, respectively. The resulting stationary solutions include the case of multi-centered solutions of extremal black holes.

Our analysis of the restrictions imposed by $N = 2$ and $N = 1$ supersymmetry on the solutions is based on the existence of a full off-shell (superconformal) multiplet calculus for $N = 2$ supergravity theories [12]. It turns out that the hypermultiplets play only a rather passive role. It proves advantageous to perform most of the analysis before writing the theory in its Poincaré form (by imposing gauge conditions or reformulating it in terms of fields that are invariant under the action of those superconformal symmetries that are absent in Poincaré supergravity). As a consequence we fix the stationary spacetime line element only at a relatively late stage of the analysis. An unusual complication is that, in order to determine the restrictions imposed by full or residual supersymmetry, it is not sufficient to consider the supersymmetry variation of the fermions only. One also needs to impose the vanishing of the supersymmetry variation of derivatives of the fermion fields. We present an argument that shows which of these variations are needed.

The paper is organized as follows. In section 2 we review the relevant features of the superconformal multiplet calculus which we use to construct $N = 2$ theories with R^2-interactions. We also briefly discuss electric/magnetic duality transformations in the presence of R^2-interactions. In section 3 we describe some of the technology needed for performing the analysis. In particular, we construct various compensating fields for S-supersymmetry (which are inert with respect to electric/magnetic duality) and we discuss a number of additional transformation laws. In section 4 we perform a detailed analysis of the restrictions imposed by $N = 2$ supersymmetry and make contact with previous results [6]. Section 5 is devoted to the analysis of the restrictions imposed by $N = 1$ supersymmetry for a particular class of embeddings of residual supersymmetry. We derive the "generalized stabilization equations" that determine the spatial dependence of the moduli fields and prove that the solutions are encoded in harmonic functions that are associated with the electric and magnetic fields. Section 6 contains our conclusions as well as an outlook. Appendices A and B explain some of the definitions and conventions used throughout this paper.

2. Action and supersymmetry transformation rules

The $N = 2$ supergravity theories that we consider are based on abelian vector multiplets and hypermultiplets coupled to the supergravity fields. The action contains the standard Einstein-Hilbert action as well as terms quadratic in the Riemann tensor. To describe such theories in a transparent way we make use of the superconformal multiplet calculus [12], which incorporates the gauge symmetries of the $N = 2$ superconformal algebra. The corresponding high degree of symmetry allows for the use of relatively small field representations. One is the Weyl multiplet, whose fields comprise the gauge fields corresponding to the superconformal symmetries and a few auxiliary fields. The other ones are abelian vector multiplets and hypermultiplets, as well as a general chiral supermultiplet, which will be treated as independent in initial stages of the analysis but at the end will be expressed in terms of the fields of the Weyl multiplet. Some of the additional (matter) multiplets will provide compensating fields which are necessary in order that the action becomes gauge equivalent to a Poincaré supergravity theory. These compensating fields bridge the deficit in degrees of freedom between the Weyl multiplet and the Poincaré supergravity multiplet. For instance, the graviphoton, represented by an abelian vector field in the Poincaré supergravity multiplet, is provided by an $N = 2$ superconformal vector multiplet.

As we will demonstrate, it is possible to analyze the conditions for residual $N = 1$ or full $N = 2$ supersymmetry directly in this superconformal setting, postponing a transition to Poincaré supergravity till the end. This implies in particular that our intermediate results are subject to local scale transformations. Only towards the end we will convert to expressions that are scale invariant.

The superconformal algebra contains general-coordinate, local Lorentz, dilatation, special conformal, chiral SU(2) and U(1), supersymmetry (Q) and special supersymmetry (S) transformations. The gauge fields associated with general-coordinate transformations (e_μ^a), dilatations (b_μ), chiral symmetry ($\mathcal{V}_\mu{}^i{}_j, A_\mu$) and Q-supersymmetry (ψ_μ^i) are realized by independent fields. The remaining gauge fields of Lorentz (ω_μ^{ab}), special conformal (f_μ^a) and S-supersymmetry transformations (ϕ_μ^i) are dependent fields. They are composite objects, which depend in a complicated way on the independent fields [12]. The corresponding curvatures and covariant fields are contained in a tensor chiral multiplet, which comprises $24 + 24$ off-shell degrees of freedom; in addition to the independent superconformal gauge fields it contains three auxiliary fields: a Majorana spinor doublet χ^i, a scalar D and a selfdual Lorentz tensor T_{abij} (where $i,j,\ldots$ are chiral SU(2) spinor indices).[1] We summarize the transformation rules for some of the independent fields of the Weyl multiplet under Q- and S-supersymmetry and under special conformal transformations, with parameters ϵ^i, η^i and $\Lambda^a_{\rm K}$, respectively,

$$
\begin{aligned}
\delta e_\mu{}^a &= \bar\epsilon^i\gamma^a\psi_{\mu i} + \text{h.c.}\,,\\
\delta\psi^i_\mu &= 2\mathcal{D}_\mu\epsilon^i - \frac{1}{8}T^{ab\,ij}\gamma_{ab}\,\gamma_\mu\epsilon_j - \gamma_\mu\eta^i\,,\\
\delta b_\mu &= \frac{1}{2}\bar\epsilon^i\phi_{\mu i} - \frac{3}{4}\bar\epsilon^i\gamma_\mu\chi_i - \frac{1}{2}\bar\eta^i\psi_{\mu i} + \text{h.c.} + \Lambda^a_{\rm K}\,e_{\mu a}\,,\\
\delta A_\mu &= \frac{1}{2}i\bar\epsilon^i\phi_{\mu i} + \frac{3}{4}i\bar\epsilon^i\gamma_\mu\chi_i + \frac{1}{2}i\bar\eta^i\psi_{\mu i} + \text{h.c.}\,,\\
\delta T^{ij}_{ab} &= 8\bar\epsilon^{[i}R(Q)^{j]}_{ab}\,,\\
\delta\chi^i &= -\frac{1}{12}\gamma_{ab}\not{D}T^{ab\,ij}\epsilon_j + \frac{1}{6}R(\mathcal{V})_{ab}{}^i{}_j\,\gamma^{ab}\epsilon^j - \frac{1}{3}iR(A)_{ab}\gamma^{ab}\epsilon^i +\\
&\quad + D\,\epsilon^i + \frac{1}{12}T^{ij}_{ab}\gamma^{ab}\eta_j\,,
\end{aligned}
\tag{2.1}
$$

where $\mathcal{D}_\mu$ are derivatives covariant with respect to Lorentz, dilatational, U(1) and SU(2) transformations, and D_μ are derivatives covariant with respect to all superconformal transformations. Throughout this paper we suppress terms of higher order in the fermions, as we will be dealing with a bosonic background. The quantities $R(Q)^i_{\mu\nu}$, $R(A)_{\mu\nu}$ and $R(\mathcal{V})_{\mu\nu}{}^i{}_j$ are supercovariant curvatures related to Q-supersymmetry, U(1) and SU(2) transformations. Their precise definitions are given in appendix B. We will make explicit use of the transformation of the S-supersymmetry gauge field,

$$
\begin{aligned}
\delta\phi^i_\mu &= -2\,f^a_\mu\gamma_a\epsilon^i + \frac{1}{4}R(\mathcal{V})_{ab}{}^i{}_j\gamma^{ab}\gamma_\mu\epsilon^j + \frac{1}{2}iR(A)_{ab}\gamma^{ab}\gamma_\mu\epsilon^i - \frac{1}{8}\not{D}T^{ab\,ij}\gamma_{ab}\gamma_\mu\epsilon_j +\\
&\quad + 2\mathcal{D}_\mu\eta^i\,.
\end{aligned}
\tag{2.2}
$$

Here f^a_μ is the gauge field of the conformal boosts, defined by (up to fermionic terms)

$$
f^{\,a}_\mu = \frac{1}{2}R(\omega,e)_\mu{}^a - \frac{1}{4}\left(D + \frac{1}{3}R(\omega,e)\right)\,e^{\,a}_\mu - \frac{1}{2}i\tilde R_\mu{}^a(A) + \frac{1}{16}T^{ij}_{\mu b}\,T^{ab}_{ij}\,,
\tag{2.3}
$$

with $R(\omega,e)_\mu{}^a = R(\omega)^{ab}_{\mu\nu}\,e^\nu_b$ the (non-symmetric) Ricci tensor and $R(\omega,e)$ the Ricci scalar. Here $R(\omega)^{ab}_{\mu\nu}$ is the curvature associated with the spin connection field ω^{ab}_μ. The spin connection is not the usual torsion-free connection, but contains the dilatational gauge field b_μ. Because of that the curvature satisfies the Bianchi identity

$$
R(\omega)^{ab}_{[\mu\nu}\,e_{\rho]\,b} = 2\partial_{[\mu}b_\nu\,e^a_{\rho]}\,.
\tag{2.4}
$$

[1]By an abuse of terminology, T_{abij} is often called the graviphoton field strength. It is antisymmetric in both Lorentz indices a,b and chiral SU(2) indices i,j. Its complex conjugate is the antiselfdual field T^{ij}_{ab}. Our conventions are such that SU(2) indices are raised and lowered by complex conjugation. The SU(2) gauge field $\mathcal{V}_\mu{}^i{}_j$ is antihermitean and traceless, i.e. $\mathcal{V}_\mu{}^i{}_j + \mathcal{V}_{\mu j}{}^i = \mathcal{V}_\mu{}^i{}_i = 0$.

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This leads to the modified pair-exchange property

$$R(\omega)_{ab}{}^{cd} - R(\omega)^{cd}{}_{ab} = 2\Big(\delta^{[c}_{[a} R(\omega,e)_{b]}{}^{d]} - \delta^{[c}_{[a} R(\omega,e)^{d]}{}_{b]}\Big)\,. \tag{2.5}$$

It is important to mention that the covariant quantities of the Weyl multiplet constitute a reduced chiral tensor multiplet, denoted by W^{abij}, whose lowest-θ component is the tensor T^{abij}. In this multiplet the superconformal gauge fields appear only through covariant derivatives and curvature tensors. From this multiplet one may form a scalar (unreduced) chiral multiplet $W^2 = [W^{abij}\,\varepsilon_{ij}]^2$, which has Weyl and chiral weights $w=2$ and $c=-2$, respectively [13].

In the following, we will allow for the presence of an arbitrary chiral background superfield [14], whose component fields will be indicated with a caret. Eventually this multiplet will be identified with W^2 in order to generate the R^2-terms in the action, but much of our analysis will not depend on this identification. We denote its bosonic component fields by $\hat A$, $\hat B_{ij}$, $\hat F^-_{ab}$ and by $\hat C$. Here $\hat A$ and $\hat C$ denote complex scalar fields, appearing at the θ^0- and θ^4-level of the chiral background superfield, respectively, while the symmetric complex SU(2) tensor $\hat B_{ij}$ and the anti-selfdual Lorentz tensor $\hat F^-_{ab}$ reside at the θ^2-level. The fermion fields at level θ and θ^3 are denoted by $\hat\Psi_i$ and $\hat\Lambda_i$. Under Q- and S-supersymmetry $\hat A$ and $\hat\Psi_i$ transform as

$$\begin{aligned}
\delta\hat A &= \bar\epsilon^i\hat\Psi_i\,,\\
\delta\hat\Psi_i &= 2\not{D}\hat A\epsilon_i + \frac{1}{2}\varepsilon_{ij}\hat F_{ab}\gamma^{ab}\epsilon^j + \hat B_{ij}\epsilon^j + 2w\hat A\eta_i\,,
\end{aligned} \tag{2.6}$$

where w denotes the Weyl weight of the background superfield. Identifying the scalar chiral multiplet with W^2 implies the following relations, which we will need later on,

$$\begin{aligned}
\hat A &= (\varepsilon_{ij}\,T^{ij}_{ab})^2\,,\\
\hat\Psi_i &= 16\,\varepsilon_{ij}R(Q)^j_{ab}\,T^{klab}\,\varepsilon_{kl}\,,\\
\hat B_{ij} &= -16\,\varepsilon_{k(i}R(\mathcal V)^k{}_{j)ab}\,T^{lmab}\,\varepsilon_{lm} - 64\,\varepsilon_{ik}\varepsilon_{jl}\bar R(Q)^k_{ab}R(Q)^{l\,ab}\,,\\
\hat F^{-ab} &= -16\,\mathcal R(M)_{cd}{}^{ab}\,T^{klcd}\,\varepsilon_{kl} - 16\,\varepsilon_{ij}\,\bar R(Q)^i_{cd}\gamma^{ab}R(Q)^{j\,cd}\,,\\
\hat\Lambda_i &= 32\,\varepsilon_{ij}\,\gamma^{ab}R(Q)^j_{cd}\,\mathcal R(M)^{cd}{}_{ab} + 16\,(\mathcal R(S)_{ab\,i} + 3\gamma_{[a}D_{b]}\chi_i)\,T^{klab}\,\varepsilon_{kl} -\\
&\quad - 64\,R(\mathcal V)_{ab}{}^k{}_i\,\varepsilon_{kl}\,R(Q)^{l\,ab}\,,\\
\hat C &= 64\,\mathcal R(M)^{-cd}{}_{ab}\,\mathcal R(M)^{-ab}_{cd} + 32\,R(\mathcal V)^{-ab\,k}{}_l\,R(\mathcal V)^{-l}_{ab\,k} -\\
&\quad - 32\,T^{ab\,ij}\,D_a\,D^cT_{cb\,ij} + 128\,\bar{\mathcal R}(S)^{ab}_i\,R(Q)^i_{ab} + 384\,\bar R(Q)^{ab\,i}\gamma_aD_b\chi_i\,.
\end{aligned} \tag{2.7}$$

We refer to appendix B for the definitions of the various curvature tensors.

Let us briefly introduce the hypermultiplets, which play a rather passive role in what follows but are needed to provide one of the compensating supermultiplets. Here we follow the presentation of [15], based on sections $A_i{}^\alpha(\phi)$ which depend on scalar fields ϕ^A, defined in the context of a so-called special hyper-Kähler space, endowed with a metric g_{AB}, and a tangent-space connection $\Gamma_A{}^\alpha{}_\beta$ as well as two covariantly constant tensors, $\bar\Omega_{\alpha\beta}$ and $G_{\alpha\bar\beta}$, which are skew-symmetric pseudo-real and hermitean, respectively. The positive and negative chirality fermions are denoted by $\zeta^{\bar\alpha}$ and ζ^α and are related by complex conjugation. The indices α and $\bar\alpha$ run over $2r$ values while the number of scalar fields labeled by indices A is equal to $4r$. Hence the special hyper-Kähler space has dimension $4r$, while the number of physical hypermultiplets will be given by $r-1$. For what follows, it suffices to consider the variations of the fermion fields ζ^α under Q- and S-supersymmetry transformations,

$$\delta\zeta^\alpha = \not{D} A_i{}^\alpha \epsilon^i - \delta\phi^B \Gamma_B{}^\alpha{}_\beta\, \zeta^\beta + A_i{}^\alpha\, \eta^i\,. \tag{2.8}$$

Here we have assumed that the hypermultiplets are neutral with respect to the gauge symmetries of the vector multiplets (to be introduced below), so there is no minimal interaction with the vector multiplet fields. The bosonic part of $D_\mu A_i{}^\alpha(\phi)$ will be given shortly.

Finally we turn to the abelian vector multiplets, labelled by an index $I = 0,1,\ldots,n$. For each value of the index I, there are $8+8$ off-shell degrees of freedom, residing in a complex scalar X^I, a doublet of chiral fermions Ω_i^I, a vector gauge field W_μ^I, and a real SU(2) triplet of auxiliary scalars Y_{ij}^I. Under Q- and S-supersymmetry the fields X^I and Ω_i^I transform as follows,

$$\begin{aligned}\delta X^I &= \bar\epsilon^i \Omega_i^I\,,\\ \delta\Omega_i^I &= 2\not{D} X^I \epsilon_i + \frac{1}{2}\varepsilon_{ij}\left(F_{\mu\nu}^{-I} - \frac{1}{4}\varepsilon_{kl} T_{\mu\nu}^{kl}\,\bar X^I\right)\gamma^{\mu\nu}\epsilon^j + Y_{ij}^I\epsilon^j + 2X^I\eta_i\,,\end{aligned} \tag{2.9}$$

where $F_{\mu\nu}^{\pm I}$ are the (anti-)selfdual parts of the vector field strength, $F_{\mu\nu}^{+I} + F_{\mu\nu}^{-I} = 2\partial_{[\mu} W_{\nu]}^I$.

The covariant quantities of the vector multiplet constitute a reduced chiral multiplet whose lowest component is the complex scalar X^I, which has Weyl and chiral weights $w=1$ and $c=-1$, respectively. A general (scalar) chiral multiplet comprises $16+16$ off-shell degrees of freedom and carries arbitrary Weyl and chiral weights. The supersymmetric action is now constructed from a chiral superspace integral of a holomorphic function of these reduced chiral multiplets. However, in order to preserve the superconformal symmetries this function must be homogeneous of second degree. This implies that its weights are $w=2$ and $c=-2$. An important observation is that this function can depend on any other chiral field, as long as its scale and chiral weights are properly accounted for. In particular, this means that we can base ourselves on a homogeneous function $F(X,\hat A)$ which is of degree two, that depends on the complex fields X^I and on the scalar of the background chiral multiplet, $\hat A$. Therefore this function satisfies the relation,

$$X^I F_I + w\hat A\, F_A = 2F\,. \tag{2.10}$$

Here F_I and F_A denote the derivatives of $F(X,\hat A)$ with respect to X^I and $\hat A$, respectively, and w denotes the weight of the background field.

In the absence of a background it is known that there are representations of the theory for which no function $F(X)$ exists, although after a suitable electric/magnetic duality transformation it can be rewritten in a form that exhibits the function $F(X)$. In the presence of a background, this feature does not seem to play a direct role, so we will simply assume the existence of $F(X,\hat{A})$. For some of the notations and background material, see [14] and the third reference of [12], where a general chiral multiplet in supergravity is discussed.

The bosonic terms of the action are encoded in the function $F(X,\hat{A})$, in the hypermultiplet sections $A_i{}^\alpha(\phi)$ and in the target space connections. They read as follows,

$$
\begin{aligned}
8\pi\, e^{-1}\,\mathcal{L} = {}& i\mathcal{D}^\mu F_I\,\mathcal{D}_\mu\bar{X}^I - iF_I\,\bar{X}^I\left(\frac{1}{6}R - D\right) - \frac{1}{8}iF_{IJ}\,Y^I_{ij}Y^{Jij} - \frac{1}{4}i\hat{B}_{ij}\,F_{AI}Y^{Iij} + \\
& + \frac{1}{4}iF_{IJ}\left(F^{-I}_{ab} - \frac{1}{4}\bar{X}^I T^{ij}_{ab}\varepsilon_{ij}\right)\left(F^{-Jab} - \frac{1}{4}\bar{X}^J T^{ijab}\varepsilon_{ij}\right) - \\
& - \frac{1}{8}iF_I\left(F^{+I}_{ab} - \frac{1}{4}X^I T_{abij}\varepsilon^{ij}\right)T^{ab}_{ij}\varepsilon^{ij} + \frac{1}{2}i\hat{F}^{-ab}\,F_{AI}\left(F^{-I}_{ab} - \frac{1}{4}\bar{X}^I T^{ij}_{ab}\varepsilon_{ij}\right) + \\
& + \frac{1}{2}iF_A\hat{C} - \frac{1}{8}iF_{AA}\left(\varepsilon^{ik}\varepsilon^{jl}\hat{B}_{ij}\hat{B}_{kl} - 2\hat{F}^-_{ab}\hat{F}^{-ab}\right) - \frac{1}{32}iF(T_{abij}\varepsilon^{ij})^2 + \text{h.c.} - \\
& - \frac{1}{2}\varepsilon^{ij}\,\bar{\Omega}_{\alpha\beta}\,\mathcal{D}_\mu A_i{}^\alpha\,\mathcal{D}^\mu A_j{}^\beta + \chi\left(\frac{1}{6}R + \frac{1}{2}D\right),
\end{aligned}
\tag{2.11}
$$

where the hyper-Kähler potential χ and the covariant derivative $\mathcal{D}_\mu A_i{}^\alpha(\phi)$ are defined by

$$
\begin{aligned}
\varepsilon_{ij}\,\chi &= \bar{\Omega}_{\alpha\beta}A_i{}^\alpha A_j{}^\beta\,, \\
\mathcal{D}_\mu A_i{}^\alpha &= \partial_\mu A_i{}^\alpha - b_\mu A_i{}^\alpha + \frac{1}{2}V_{\mu i}{}^j\,A_j{}^\alpha + \partial_\mu\phi^A\,\Gamma_A{}^\alpha{}_\beta\,A_i{}^\beta\,.
\end{aligned}
\tag{2.12}
$$

Even in the presence of the chiral background the lagrangian has the form of a generalized Maxwell lagrangian with terms that are at most quadratic in the field strengths. This feature will change once we start eliminating auxiliary fields.[2] Hence it is advisable to first solve the Maxwell equations, before eliminating the auxiliary fields. One distinguishes the Bianchi equations, which are expressed directly in terms of the field strengths $F^{\pm I}_{\mu\nu}$, and the equations for the electric and magnetic "displacement" fields $G^\pm_{\mu\nu I}$, which are proportional to the variation of the action with respect

[2] Because the chiral background field given in (2.7) involves terms of higher order in derivatives, the lagrangian will contain higher-derivative interactions. The most conspicuous ones are the interactions quadratic in the Riemann curvature. Such lagrangians generically describe negative-metric states. However, they should not be regarded as elementary lagrangians, but rather as effective lagrangians. This implies that auxiliary fields that appear with derivatives, should still be eliminated. This leads to an infinite series of terms that corresponds to an expansion in terms of momenta divided by the Planck mass.

to the $F^{\pm I}_{\mu\nu}$. With suitable proportionality factors, these tensors read (we suppress fermion contributions),

$$G^+_{\mu\nu I} = \bar F_{IJ} F^{+J}_{\mu\nu} + \mathcal{O}^+_{\mu\nu I}\,, \qquad G^-_{\mu\nu I} = F_{IJ} F^{-J}_{\mu\nu} + \mathcal{O}^-_{\mu\nu I}\,, \tag{2.13}$$

where

$$\begin{aligned} \mathcal{O}^+_{\mu\nu I} &= \frac{1}{4}(F_I - \bar F_{IJ} X^J)\, T_{\mu\nu ij}\varepsilon^{ij} + \hat F^+_{\mu\nu}\, \bar F_{IA}\,, \\ \mathcal{O}^-_{\mu\nu I} &= \frac{1}{4}(\bar F_I - F_{IJ} \bar X^J)\, T^{ij}_{\mu\nu}\, \varepsilon_{ij} + \hat F^-_{\mu\nu}\, F_{IA}\,. \end{aligned} \tag{2.14}$$

In terms of these tensors the Maxwell equations in the absence of charges read (in the presence of the background), $\mathcal{D}^a(F^- - F^+)^I_{ab} = 0$, and $\mathcal{D}^a(G^- - G^+)_{ab\,I} = 0$. Eventually we will solve these equations for a given configuration of electric and magnetic charges in a stationary geometry. These charges will be denoted by (p^I, q_J) and are normalized such that for a stationary multi-centered solution with charges at centers $\vec x_A$, Maxwell's equations read

$$\partial_\mu \begin{pmatrix} \sqrt{g}(F^- - F^+)^{I\,\mu t} \\ \sqrt{g}(G^- - G^+)^{\mu t}_I \end{pmatrix} = 4i\pi \sum_A \delta(\vec x - \vec x_A) \begin{pmatrix} p^I_A \\ q_{AI} \end{pmatrix}. \tag{2.15}$$

Observe that $\sqrt{g}\,(F^- - F^+)^{I\,\mu\nu}$ and $\sqrt{g}\,(G^- - G^+)^{\mu\nu}_I$ are Weyl invariant quantities.

The field equations of the vector multiplets are subject to equivalence transformations corresponding to electric/magnetic duality, which do not affect the fields of the Weyl multiplet and of the chiral background. As is well known, the following two complex $(2n+2)$-component vectors transform linearly under the $\mathrm{SP}(2n+2;\mathbb{R})$ duality group,

$$\begin{pmatrix} X^I \\ F_I(X,\hat A) \end{pmatrix} \qquad \text{and} \qquad \begin{pmatrix} F^{\pm I}_{ab} \\ G^{\pm}_{ab\,I} \end{pmatrix}, \tag{2.16}$$

but more such vectors can be constructed. The first vector has weights $w = 1$ and $c = -1$, whereas the second one has zero Weyl and chiral weights. From (2.15) and (2.16) it follows that also the charges (p^I, q_J) comprise a symplectic vector. In the presence of these charges the symplectic transformations are restricted to an integer-valued subgroup that keeps the lattice of electric/magnetic charges invariant.

The electric/magnetic duality transformations cannot be performed at the level of the action, but only at the level of the equations of motion. After applying the transformations one can find the corresponding action. This is then characterized by a relation between two different functions $F(X,\hat A)$. While the background field $\hat A$ is inert under the dualities, it nevertheless enters in the explicit form of the transformations. For a discussion of this phenomenon and its consequences, see [14].

The various transformation rules only take a symplectically invariant form when one solves the field equations for the auxiliary fields Y^I_{ij} [14],

$$Y^I_{ij} = iN^{IJ}\left(F_{JA}\, \hat B_{ij} - \bar F_{JA}\, \varepsilon_{ik}\varepsilon_{jl}\, \hat B^{kl}\right). \tag{2.17}$$

With this result we can cast $\delta\Omega_i^I$ and $\delta\hat{\Psi}_i$ in a symplectically covariant form (we suppress fermionic bilinears),

$$\begin{aligned}\begin{pmatrix}\delta\Omega_i^I\\ \delta(F_{IJ}\Omega_i^J+F_{IA}\hat{\Psi}_i)\end{pmatrix} &= 2\not{D}\begin{pmatrix}X^I\\ F_I\end{pmatrix}\epsilon_i+\frac{1}{2}\varepsilon_{ij}\gamma^{ab}\epsilon^j\left[\begin{pmatrix}F_{ab}^{-I}\\ G_{abI}^{-}\end{pmatrix}-\frac{1}{4}\varepsilon_{kl}T_{ab}^{kl}\begin{pmatrix}\bar{X}^I\\ \bar{F}_I\end{pmatrix}\right]+\\ &\quad+i\hat{B}_{ij}\,\epsilon^j\begin{pmatrix}N^{IJ}F_{JA}\\ \bar{F}_{IJ}N^{JK}F_{KA}\end{pmatrix}-\\ &\quad-i\varepsilon_{ik}\varepsilon_{jl}\hat{B}^{kl}\,\epsilon^j\begin{pmatrix}N^{IJ}\bar{F}_{JA}\\ F_{IJ}N^{JK}\bar{F}_{KA}\end{pmatrix}+2\eta_i\begin{pmatrix}X^I\\ F_I\end{pmatrix}.\end{aligned} \tag{2.18}$$

In the above formulae, N^{IJ} is the inverse of

$$N_{IJ}=-iF_{IJ}+i\bar{F}_{IJ}\,. \tag{2.19}$$

3. More supersymmetry variations

In the superconformal tensor calculus two of the matter supermultiplets are required in order to provide the compensating degrees of freedom that are essential for making the system equivalent to a Poincaré supergravity theory. One of these multiplets is always a vector multiplet and for the second one we choose a hypermultiplet. This implies that the number of physical vector multiplets is equal to n and the number of physical hypermultiplets is equal to $r-1$.

In this section we will evaluate the supersymmetry variations of a number of spinors that are needed in the analysis in subsequent sections. The results of this section follow from those given in the previous one. Some of the spinors can act as suitable compensating fields with regard to S-supersymmetry. We also evaluate the supersymmetry variations of the supercovariant derivative of the spinors belonging to one of the matter multiplets as well as the variation of the supersymmetry field strength $R(Q)^i_{ab}$. This analysis naturally leads us to the definition of a number of bosonic quantities that play a central role in what follows.

The first spinor we consider is expressed in terms of hypermultiplet fermions and reads

$$\zeta_i^{\rm H}\equiv\chi^{-1}\bar{\Omega}_{\alpha\beta}A_i{}^\alpha\,\zeta^\beta\,. \tag{3.1}$$

Its supersymmetry variation reads

$$\delta\zeta_i^{\rm H}=\chi^{-1}\bar{\Omega}_{\alpha\beta}A_i{}^\alpha\not{D}A_j{}^\beta\,\epsilon^j+\varepsilon_{ij}\eta^j\,, \tag{3.2}$$

where χ is the hyper-Kähler potential defined in (2.12) and where terms proportional to the fermion fields are suppressed. It is important to realize that one has the decomposition [15]

$$\chi^{-1}\bar{\Omega}_{\alpha\beta}A_i{}^\alpha D_\mu A_j{}^\beta=\frac{1}{2}k_\mu\,\varepsilon_{ij}+k_{\mu\,ij}\,, \tag{3.3}$$

where k_μ is real and given by

$$k_\mu = \chi^{-1}(\partial_\mu - 2\, b_\mu)\chi\,, \tag{3.4}$$

and $k_{\mu\, ij}$ is symmetric in i, j and pseudoreal so that it transforms as a vector under SU(2). Its explicit form is not important for us. Hence we write

$$\delta\zeta_i^{\rm H} = \frac{1}{2}\not k\; \varepsilon_{ij}\, \epsilon^j + \not k_{ij}\, \epsilon^j + \varepsilon_{ij}\, \eta^j\,. \tag{3.5}$$

In the vector multiplet sector there are two spinors that can be constructed which transform as scalars under electric/magnetic duality. One, denoted by ζ_i^V, transforms inhomogeneously under S-supersymmetry. It can be conveniently introduced from the variation of the symplectically invariant expression (with $w=2$ and $c=0$)

$$\mathrm{e}^{-\mathcal{K}} = i\Big[\bar X^I\, F_I(X, \hat A) - \bar F_I(\bar X, \bar{\hat A})\, X^I\Big]\,. \tag{3.6}$$

Here $\mathcal{K}$ resembles the Kähler potential in special geometry. Its supersymmetry variation leads to the spinor

$$\zeta_i^{\rm V} \equiv -\Big(\Omega_i^I\, \frac{\partial}{\partial X^I} + \hat\Psi_i\, \frac{\partial}{\partial \hat A}\Big)\mathcal{K} = -i\, \mathrm{e}^{\mathcal{K}} \Big[(\bar F_I - \bar X^J F_{IJ})\Omega_i^I - \bar X^I F_{IA}\, \hat\Psi_i\Big]\,. \tag{3.7}$$

It is obvious that $\zeta_i^{\rm V}$ transforms as a scalar under symplectic reparameterizations, because it follows from a symplectic scalar. This can also been seen by noting that $\zeta_i^{\rm V}$ is generated by the symplectic product $\bar F_I\, \delta X^I - \bar X^I\, \delta F_I$. This leads us to introduce yet another spinor ζ_i^0 generated by $F_I\, \delta X^I - X^I\, \delta F_I$,

$$\zeta_i^0 \equiv (F_I - X^J F_{IJ})\Omega_i^I - X^I F_{IA}\, \hat\Psi_i\,. \tag{3.8}$$

This spinor is invariant under S-supersymmetry and it vanishes in the absence of the chiral scalar background field. However, it does not play a useful role in what follows.

Under Q- and S-supersymmetry $\zeta_i^{\rm V}$ transforms as

$$\begin{aligned}\delta\zeta_i^{\rm V} = {}& \mathrm{e}^{\mathcal{K}}\not{D}\mathrm{e}^{-\mathcal{K}}\epsilon_i + 2i\not{\mathcal{A}}\; \epsilon_i - \frac{1}{2}i\varepsilon_{ij}\, \mathcal{F}^-_{ab}\, \gamma^{ab}\epsilon^j + \\ & + \mathrm{e}^{\mathcal{K}}\, N^{IJ}\Big[(\bar F_I - \bar F_{IK}\bar X^K)F_{JA}\, \hat B_{ij} - (\bar F_I - F_{IK}\bar X^K)\bar F_{JA}\, \varepsilon_{ik}\varepsilon_{jl}\hat B^{kl}\Big]\, \epsilon^j + 2\, \eta_i\,,\end{aligned} \tag{3.9}$$

where we ignored higher-order fermionic terms. The quantity $\mathcal{A}_\mu$ resembles a covariantized (real) Kähler connection and $\mathcal{F}^-_{ab}$ is an anti-selfdual tensor,

$$\begin{aligned}\mathcal{A}_\mu &= \frac{1}{2}\mathrm{e}^{\mathcal{K}}\Big(\bar X^J\, \overset{\leftrightarrow}{\mathcal{D}}_\mu\, F_J - \bar F_J\, \overset{\leftrightarrow}{\mathcal{D}}_\mu\, X^J\Big)\,,\\ \mathcal{F}^-_{ab} &= \mathrm{e}^{\mathcal{K}}\, \Big(\bar F_I\, F^{-I}_{ab} - \bar X^I\, G^-_{ab\, I}\Big)\,.\end{aligned} \tag{3.10}$$

There is another symplectically invariant contraction of the field strengths,

$$\mathrm{e}^{\mathcal{K}}\, \Big(F_I\, F^{-I}_{ab} - X^I\, G^-_{ab\, I}\Big) + \frac{1}{4}i\varepsilon_{ij}T^{ij}_{ab} = \mathrm{e}^{\mathcal{K}}\, F_{IA}\Big[w\hat A\, \Big(F^{-I}_{ab} - \frac{1}{4}\bar X^I\, \varepsilon_{ij}T^{ij}_{ab}\Big) - X^I\, \hat F^-_{ab}\Big], \tag{3.11}$$

which appears in the variation of ζ_i^0.

As it turns out we also need to consider the supersymmetry variations of derivatives of the fermion fields. However, one can restrict oneself to the variation of the supercovariant derivative of a single fermion field, as we will discuss in the next section. For this field we choose $\zeta_i^{\rm H}$, for which we present the variation under Q- and S-supersymmetry,

$$\begin{aligned}\delta(D_\mu\zeta_i^{\rm H}) = {}& \frac{1}{2}\mathcal{D}_\mu(\chi^{-1}\mathcal{D}_\nu\chi)\,\varepsilon_{ij}\,\gamma^\nu\epsilon^j + \mathcal{D}_\mu k_{\nu ij}\,\gamma^\nu\epsilon^j - \\ & -\frac{1}{32}\chi^{-1/2}(\delta_i^j\mathcal{D}_\nu - k_{\nu ik}\varepsilon^{kj})(\chi^{1/2}T_{ab}^{lm}\varepsilon_{lm})\,\gamma^\nu\gamma^{ab}\gamma_\mu\epsilon_j + \\ & +\varepsilon_{ij}\left[f_\mu{}^a\gamma_a\epsilon^j - \frac{1}{8}R(\mathcal{V})^j{}_{kab}\gamma^{ab}\gamma_\mu\epsilon^k - \frac{1}{4}iR(A)_{ab}\gamma^{ab}\gamma_\mu\epsilon^j\right] + \\ & +\left(\frac{1}{4}\chi^{-1}\not{\mathcal{D}}\chi\,\varepsilon_{ij} + \frac{1}{2}\not{k}_{ij}\right)\,\gamma_\mu\eta^j\,. \end{aligned} \tag{3.12}$$

Finally we present the variation of the curvature tensor $R(Q)_{\mu\nu}^i$, defined by

$$R(Q)_{\mu\nu}^i = 2\mathcal{D}_{[\mu}\psi_{\nu]}^i - \gamma_{[\mu}\phi_{\nu]}^i - \frac{1}{8}T_{ab}^{ij}\gamma^{ab}\gamma_{[\mu}\psi_{\nu]j}\,, \tag{3.13}$$

where ϕ_μ^i is the dependent gauge field associated with S-supersymmetry, defined in appendix B. The variation of this tensor reads,

$$\delta R(Q)_{ab}^i = -\frac{1}{2}\not{\mathcal{D}}T_{ab}^{ij}\,\epsilon_j + R(\mathcal{V})_{ab}^-{}^i{}_j\,\epsilon^j - \frac{1}{2}\mathcal{R}(M)_{ab}{}^{cd}\,\gamma_{cd}\epsilon^i + \frac{1}{8}T_{cd}^{ij}\,\gamma^{cd}\gamma_{ab}\,\eta_j\,, \tag{3.14}$$

where $\mathcal{R}(M)_{ab}{}^{cd}$ is defined in appendix B.

4. Fully supersymmetric field configurations

From the supersymmetry variations presented in the previous two sections one can determine the conditions on the bosonic fields imposed by the requirement of full $N=2$ supersymmetry. These conditions follow from setting all Q-supersymmetry variations of the fermionic quantities to zero. However, these variations are determined up to an S-supersymmetry transformation. Thus one can either impose the vanishing of all Q-variations up to a uniform S-supersymmetry transformation, or one can restrict oneself to linear combinations that are invariant under S-supersymmetry and require their Q-supersymmetry variations to vanish. Examples of such S-invariant combinations are, for instance, $\Omega_i^I - X^I\zeta_i^{\rm V}$ and $\hat\Psi_i - w\hat A\,\zeta_i^{\rm V}$, while the spinor ζ_i^0 is S-invariant by itself. In this section we will include an arbitrary number of hypermultiplets.

We start by considering the S-supersymmetric linear combination of $\zeta_i^{\rm V}$ and $\zeta_i^{\rm H}$. Requiring its Q-supersymmetry variation to vanish for all supersymmetry parameters, we establish immediately that

$$\mathcal{F}_{ab}^- = \hat B_{ij} = k_{\mu\,ij} = \mathcal{A}_\mu = 0\,, \tag{4.1}$$

and

$$\mathcal{D}_\mu\left(\mathrm{e}^{\mathcal{K}}\chi\right) = 0\,. \tag{4.2}$$

Comparing the supersymmetry variations of the vector multiplet fermions to those of ζ_i^{V} leads to

$$\begin{aligned}
F_{ab}^{-I} &= \frac{1}{4}\varepsilon_{kl}T_{ab}^{kl}\,\bar{X}^I\,,\\
G_{abI}^{-} &= \frac{1}{4}\varepsilon_{kl}T_{ab}^{kl}\,\bar{F}_I\,,\\
\mathcal{D}_\mu\left(\mathrm{e}^{\mathcal{K}/2}X^I\right) &= \mathcal{D}_\mu\left(\mathrm{e}^{\mathcal{K}/2}F_I\right) = 0\,.
\end{aligned} \tag{4.3}$$

These equations themselves again imply that $\mathcal{F}_{ab}^{-}$ and $\mathcal{A}_\mu$ vanish. Furthermore, by using the explicit expression of the tensors G_{abI}^{-}, one finds that $\hat{F}_{ab}^{-} = 0$. The last two equations imply that we also have

$$\mathcal{D}_\mu\left(\mathrm{e}^{w\mathcal{K}/2}\hat{A}\right) = 0\,. \tag{4.4}$$

From the supersymmetry variations of the hypermultiplets we find a similar result,

$$\mathcal{D}_\mu\left(\chi^{-1/2}A_i{}^\alpha\right) = 0\,. \tag{4.5}$$

Observe that all the above equations are K-invariant.

Subsequently we compare the supersymmetry variations of the spinors χ^i and ζ_i^{V}, which leads to the relations,

$$D = R(\mathcal{V})_{ab}{}^i{}_j = R(A)_{ab} = \mathcal{D}_a\left(\mathrm{e}^{-\mathcal{K}/2}T^{abij}\right) = 0\,. \tag{4.6}$$

With these results it follows that the vector field strengths satisfy the following equations,

$$\mathcal{D}^a F_{ab}^{-I} = \mathcal{D}^a G_{abI}^{-} = 0\,, \tag{4.7}$$

which imply (but are stronger than) the equations of motion and the Bianchi identities for the vector fields.

A similar calculation for the curvature $R(Q)_{ab}^i$ yields

$$\begin{aligned}
\mathcal{D}_c T_{ab}^{ij} &= -\frac{1}{2}\mathcal{D}_d\mathcal{K}\left(\delta_c^d\,T_{ab}^{ij} - 2\delta_{[a}^d\,T_{b]c}^{ij} + 2\eta_{c[a}\,T_{b]}^{ij\,d}\right),\\
\mathcal{R}(M)_{ab}{}^{cd} &= 0\,.
\end{aligned} \tag{4.8}$$

The first equation is consistent with the result found earlier. Because $D = 0$, $\mathcal{R}(M)_{ab}{}^{cd}$ is just the traceless part of the curvature tensor $R(\omega)_{ab}{}^{cd}$ associated with the spin connection field ω_μ^{ab} (which at this stage depends on the dilatational gauge field b_μ). Upon suppressing b_μ, this tensor becomes equal to the Weyl tensor. Hence the above condition will eventually lead to the conclusion that $N = 2$ supersymmetric solutions require a conformally flat spacetime. We stress again that all of the above conditions are K-invariant.

Before continuing, let us make a few remarks. First of all, we note that at this stage all equations are consistent with all the superconformal symmetries; in particular, we have not yet fixed a scale. All the above results are also manifestly consistent with electric/magnetic duality. Secondly we found a number of conditions on the chiral background field, namely $\hat{B}_{ij} = \hat{F}^-_{ab} = 0$ and the covariant constancy of $\exp(w\mathcal{K}/2)\hat{A}$. So far no conditions have been derived for its highest-θ component $\hat{C}$, but by considering the supersymmetry variation of the spinor $\hat{\Lambda}_i$ one can easily show that $\hat{C} = 0$. It is illuminating to verify whether these results hold for the chiral field starting with $\hat{A} = [T^{abij}\varepsilon_{ij}]^2$. It turns out that they are indeed satisfied on the basis of the above results, with the exception of the $\hat{C}$ component which contains a term proportional to the second derivative of $T_{ab\,ij}$. Also in view of later applications we consider this term in more detail and note that the bosonic contribution to the second derivative of T^{ij}_{ab} takes the form

$$D_\mu D_c T^{ij}_{ab} = \mathcal{D}_\mu \mathcal{D}_c T^{ij}_{ab} + f_{\mu c}\, T^{ij}_{ab} - 2 f_{\mu[a}\, T^{ij}_{b]c} + 2 f_\mu{}^d\, \eta_{c[a}\, T^{ij}_{b]d} \,. \tag{4.9}$$

Consequently

$$D_\mu D^a T^{ij}_{ab} = \mathcal{D}_\mu \mathcal{D}^a T^{ij}_{ab} - f_\mu{}^a\, T^{ij}_{ab} \,. \tag{4.10}$$

With this result we consider the relevant term in $\hat{C}$,

$$T^{ab\,ij}\, D_a D^c T_{cb\,ij} = T^{ab\,ij}\, \mathcal{D}_a \mathcal{D}^c T_{cb\,ij} - f_a{}^c\, T^{ab\,ij}\, T_{cb\,ij} \,, \tag{4.11}$$

where we note in passing that, in the first term on the right-hand side, we can symmetrize the derivatives as the antisymmetric part vanishes due to the (anti-)self-duality of the T-fields. By using the equations found above, we can work out the double derivative on the T-field, and verify whether it vanishes against the second term proportional to $f_\mu{}^a$.

Rather than determining $f_\mu{}^a$ in this way, we continue and consider the supersymmetry variation of the supercovariant derivatives of fermion fields. First we make the observation that the derivatives of S-invariant combinations of fields, whose Q-supersymmetric variations were already required to vanish in the bosonic background, will still vanish. But we can also compare the variation of the supercovariant derivative of a fermion field to the variation of a fermion field without derivatives. Consider for example the Q-variation of the following S-invariant expression

$$D_\mu \zeta^{\mathrm{H}}_i + \left(-\frac{1}{4}\chi^{-1} \not{\mathcal{D}}\chi\, \delta_i^j + \frac{1}{2} \not{k}_{ik}\varepsilon^{kj} \right) \gamma_\mu \zeta^{\mathrm{H}}_j \,. \tag{4.12}$$

The derivative of another fermion field can now be written as the derivative of an S-invariant linear combination of that fermion field with a bosonic expression times ζ^{H}_i, which is one of the previously considered linear combinations whose vanishing variation in the supersymmetric background has already been ensured, a term proportional to (4.12) and terms proportional to $\zeta_i^{\rm H}$ without a derivative. Therefore, once we have imposed that the variation of (4.12) vanishes, then the variation of the derivative of every other fermion field is guaranteed to vanish against some bosonic term times the variation of $\zeta_i^{\rm H}$. Consequently variations of such linear combinations can be ignored and our only task is to require that the variation of (4.12) vanishes. Note that the above argument can be extended to variations of multiple derivatives as well, which therefore can also be ignored.

Imposing the condition that the Q-supersymmetry variation of (4.12) vanishes, we find that most terms vanish already by virtue of previous results and we are left with just one more equation,

$$D_\mu\Big(\chi^{-1}D^a\chi\Big) = \frac{1}{2}\Big(\chi^{-1}\,D_\mu\chi\Big)\Big(\chi^{-1}D^a\chi\Big) - \frac{1}{4}e_\mu^{\;a}\,\Big(\chi^{-1}\,D_c\chi\Big)^2 . \tag{4.13}$$

Note that we have superconformal derivatives here which involve the gauge field $f_\mu{}^a$ associated with conformal boosts. Upon using the previous results (4.2), (4.3) and (4.5), all equations coincide. Hence we are left with the following equation for $f_\mu^{\;a}$,

$$f_\mu{}^a = -\frac{1}{2}\mathcal{D}_\mu\Big(\mathrm{e}^{\mathcal{K}}\,\mathcal{D}^a\mathrm{e}^{-\mathcal{K}}\Big) + \frac{1}{4}\Big(\mathrm{e}^{\mathcal{K}}\,\mathcal{D}_\mu\mathrm{e}^{-\mathcal{K}}\Big)\Big(\mathrm{e}^{\mathcal{K}}\,\mathcal{D}^a\mathrm{e}^{-\mathcal{K}}\Big) - \frac{1}{8}e_\mu^{\;a}\,\Big(\mathrm{e}^{\mathcal{K}}\,\mathcal{D}_c\mathrm{e}^{-\mathcal{K}}\Big)^2 , \tag{4.14}$$

which is K-invariant. With this result we can verify that the term (4.11) vanishes as well, so that we establish that the $\hat{C}$ component of the Weyl multiplet vanishes. The above equation (4.14) can be rewritten as

$$R(\omega,e)_\mu{}^a - \frac{1}{6}R(\omega,e)\,e_\mu^{\;a} = -\frac{1}{8}T^{ij}_{\mu b}\,T^{ab}_{ij} + \mathcal{D}_\mu\mathcal{D}^a\mathcal{K} + \frac{1}{2}\mathcal{D}_\mu\mathcal{K}\,\mathcal{D}^a\mathcal{K} - \frac{1}{4}e_\mu^{\;a}\,(\mathcal{D}_c\mathcal{K})^2 . \tag{4.15}$$

So far the analysis is valid for any chiral background field. For the rest of this section we assume that the chiral multiplet is given by (2.7) so that at this point we have identified all supersymmetric configurations in the presence of R^2-terms. The results obtained so far are in a manifestly conformally covariant form. We can now impose gauge choices and set $b_\mu = 0$ (because of the K-invariance the conditions found above are in fact independent of b_μ) and $\exp[\mathcal{K}]$ equal to a constant. (Alternative we may use $\exp[\mathcal{K}]$ as a compensator to make all quantities invariant under scale transformations, at which point the field b_μ will drop out.) The values of $\exp[-\mathcal{K}]$ and χ are related. With the choice that we made for the action we find that $\chi = -2\exp[-\mathcal{K}]$ as a result of the field equation for the field D. For future reference, we give both the field equations for the field D and for the U(1) gauge field A_μ,

$$\begin{aligned} 3\,\mathrm{e}^{-\mathcal{K}} + \frac{1}{2}\chi = {} & -192i\,D(F_A - \bar{F}_A) + \\ & + 4i\Big\{(\varepsilon_{ij}\,T^{ij}_{cd})^{-2}\,\varepsilon_{kl}T^{abkl}\,(F_I\,F^{-I}_{ab} - X^I\,G^-_{abI}) - \text{h.c.}\Big\} , \end{aligned} \tag{4.16}$$

$$
\begin{aligned}
\mathrm{e}^{-\mathcal{K}}\mathcal{A}_a &= 128i\,\mathcal{D}^b\Big(F_A\,R(A)^-_{ab} - \mathrm{h.c.}\Big) - 8\,\mathcal{D}_c(F_A+\bar F_A)\,T_{ijab}\,T^{ijcb} + \\
&\quad + 8(F_A-\bar F_A)\,\Big(T^{ij}_{ab}\mathcal{D}_c T^{cb}_{ij} - T_{ijab}\,\mathcal{D}_c T^{cbij}\Big) - \\
&\quad - 8\,\mathcal{D}^b\Big\{(\varepsilon_{ij}\,T^{ij}_{de})^{-2}\,\varepsilon_{kl}T^{kl\,c}{}_{[a}\,(F_I\,F^{-I}_{b]c} - X^I\,G^-_{b]cI}) + \mathrm{h.c.}\Big\}\,. \qquad (4.17)
\end{aligned}
$$

Observe that these field equations can only be found from the action, and cannot be obtained from requiring that the supersymmetry variations vanish, because the action consists of a linear combination of two actions that are separately invariant, corresponding to the vector multiplets and the hypermultiplets, respectively (we point out that the hypermultiplets contribute only fermionic terms to (4.17), which have been suppressed above). The coefficient of the Ricci scalar in the action is now equal to $-(16\pi)^{-1}\exp[-\mathcal{K}]$, so that Newton's constant equals $G_{\mathrm{N}} = \exp[\mathcal{K}]$, assuming a conventionally normalized flat metric. Furthermore we can put the gauge fields A_μ and $\mathcal{V}_\mu{}^i{}_j$ to zero, because their field strengths vanish.

The most general $N=2$ supersymmetric background can now be characterized as follows. First of all the spacetime has zero Weyl tensor and is thus conformally flat. Its Ricci tensor is given by

$$
R_{\mu\nu} = -\frac{1}{8}T^{ij}_{\mu\rho}\,T_{ij\nu}{}^\rho\,, \qquad (4.18)
$$

where $T_{ij\mu\nu}$ ($T^{ij}_{\mu\nu}$) is a covariantly constant (anti-)selfdual tensor. Furthermore we have a number of constants X^I. The electric/magnetic field strengths are also covariantly constant and given by

$$
F^{-I}_{\mu\nu} = \frac{1}{4}\varepsilon_{kl}T^{kl}_{\mu\nu}\,\bar X^I\,, \qquad G^-_{\mu\nu I} = \frac{1}{4}\varepsilon_{kl}T^{kl}_{\mu\nu}\,\bar F_I\,. \qquad (4.19)
$$

By using relations for products of (anti-)selfdual tensors one can verify that the integrability condition that follows from the covariant constancy of the tensor fields $T^{ij}_{\mu\nu}$, is identically satisfied. In order to investigate explicit solutions one chooses coordinates such that the metric reads

$$
g_{\mu\nu} = \mathrm{e}^{2f(x)+\mathcal{K}}\,\eta_{\mu\nu}\,, \qquad (4.20)
$$

with $\eta_{\mu\nu}$ the flat Minkowski metric (normalized in the standard way). We included the factor $\exp[\mathcal{K}]$, which we adjusted to a constant, so that the function f is independent of the scale. To have a vanishing Ricci scalar the function $\exp[f]$ must be harmonic,

$$
\eta^{\mu\nu}\partial_\mu\partial_\nu\,\mathrm{e}^f = 0\,. \qquad (4.21)
$$

The remaining conditions are (here we raise and lower indices with the flat metric)

$$
\begin{aligned}
R_{\mu\nu} &= 2\partial_\mu\partial_\nu f - 2\partial_\mu f\,\partial_\nu f + \eta_{\mu\nu}\,(\partial_\rho f)^2 = -\frac{1}{8}T^{ij}_{\mu\rho}\,T_{ij\nu}{}^\rho\,\mathrm{e}^{-2f-\mathcal{K}}\,, \\
\partial_\mu T^{ij}_{\nu\rho} &= 2\partial_\mu f\,T^{ij}_{\nu\rho} - 2\partial_{[\nu}f\,T^{ij}_{\rho]\mu} + 2\eta_{\mu[\nu}\,T^{ij}_{\rho]\sigma}\,\partial^\sigma f\,. \qquad (4.22)
\end{aligned}
$$

As a result of the second condition we derive

$$\partial_{[\mu} T^{ij}_{\nu\rho]} = \partial^{\mu} T^{ij}_{\mu\nu} = 0\,, \tag{4.23}$$

so that $T^{ij}_{\mu\nu}$ is a harmonic tensor.

We are interested in time-independent solutions so that we assume that f is independent of the time t. In that case we can express the tensor field in terms of a complex potential Φ. Denoting spatial world indices by $\hat{a}, \hat{b}, \hat{c}$, we may write

$$\varepsilon_{ij} T^{ij}_{\hat{a}\hat{b}} = \varepsilon_{\hat{a}\hat{b}\hat{c}}\, \partial_{\hat{c}} \Phi\,, \qquad \varepsilon_{ij} T^{ij}_{t\hat{a}} = i \partial_{\hat{a}} \Phi\,, \tag{4.24}$$

where Φ is a complex harmonic function. The equations are now solved for by

$$\Phi = 4\, z\, \mathrm{e}^{f + \mathcal{K}/2}\,, \tag{4.25}$$

with z a constant phase factor, and f satisfying

$$\mathrm{e}^{f}\, \partial_{\hat{a}} \partial_{\hat{b}} \mathrm{e}^{f} = 3\, \partial_{\hat{a}} \mathrm{e}^{f}\, \partial_{\hat{b}} \mathrm{e}^{f} - \delta_{\hat{a}\hat{b}}\, (\partial_{\hat{c}} \mathrm{e}^{f})^2\,. \tag{4.26}$$

This system of differential equations can be integrated. Its solution is unique (up to translations) and is given by $\exp[f(r)] = c/r$, where c is a real constant. This leads to a Bertotti-Robinson spacetime, the geometry of which describes the near-horizon limit of an extremal black hole with horizon at $r = 0$. Thus there exist no fully supersymmetric multi-centered solutions, which is not suprising in view of the fact that the differential equations (4.26) are non linear in $\exp[f]$. The field $\hat{A}$ is now equal to

$$\hat{A} = (\varepsilon_{ij} T^{ij}_{ab})^2 = \frac{64\ \mathrm{e}^{-\mathcal{K}}}{\bar{z}^2\, \mathrm{e}^{2f(r)}}\, (\partial_{\hat{a}} f)^2\,. \tag{4.27}$$

From evaluating (4.19) it follows that the electric and magnetic charges are equal to

$$p^I = c\, \mathrm{e}^{\mathcal{K}/2}\, [\bar{z}\, X^I + z\, \bar{X}^I]\,, \qquad q_I = c\, \mathrm{e}^{\mathcal{K}/2}\, [\bar{z}\, F_I + z\, \bar{F}_I]\,. \tag{4.28}$$

With this result we consider the so-called BPS mass, which takes the form

$$Z = \mathrm{e}^{\mathcal{K}/2} (p^I\, F_I - q_I\, X^I) = -iz\, c\,, \tag{4.29}$$

so that we obtain the equations (sometimes called stabilization equations) [1, 2],

$$\bar{Z} \begin{pmatrix} X^I \\ F_I \end{pmatrix} - Z \begin{pmatrix} \bar{X}^I \\ \bar{F}_I \end{pmatrix} = i\, \mathrm{e}^{-\mathcal{K}/2} \begin{pmatrix} p^I \\ q_I \end{pmatrix}. \tag{4.30}$$

Observe that this result is covariant with respect to electric/magnetic duality.

Finally we note that the area in Planck units equals

$$\frac{\text{Area}}{G_{\mathrm{N}}} = 4\pi\, c^2 = 4\pi\, |Z|^2\,. \tag{4.31}$$

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This does not determine the black hole entropy, because the Bekenstein-Hawking area law is not applicable for these black holes [10]. After including an appropriate correction one obtains instead [6]

$$\mathcal{S} = \pi \Big[|Z|^2 - 256 \, \mathrm{Im}[F_A(X^I, \hat{A})] \Big] \,, \tag{4.32}$$

where $\hat{A} = -64 \, \bar{Z}^{-2} \, \mathrm{e}^{-\mathcal{K}}$.

In section 5 we will be using another coordinate frame with line element given by

$$\mathrm{d}s^2 = -\mathrm{e}^{2g} \, \mathrm{d}t^2 + \mathrm{e}^{-2g} \, \mathrm{d}\vec{x}^2 \,. \tag{4.33}$$

The conformal coordinates of this section are related to those of the above frame by

$$t \longrightarrow \frac{d}{c^2 \, \mathrm{e}^{\mathcal{K}}} \, t \,, \qquad \vec{x} \longrightarrow d \, \frac{\vec{x}}{|\vec{x}|^2} \,, \tag{4.34}$$

where d is some real constant. The function e^{-2g} in (4.33) corresponding to the line element (4.20) is equal to

$$\mathrm{e}^{-2g} = \frac{c^2 \, \mathrm{e}^{\mathcal{K}}}{|\vec{x}|^2} \,. \tag{4.35}$$

For later reference let us give the field strengths (4.19) in the frame (4.33),

$$F^{-\,I}_{tm} = iz\bar{X}^I \, \mathrm{e}^g \, \frac{x^m}{|\vec{x}|^2} \,, \qquad G^-_{tm\,I} = iz\bar{F}_I \, \mathrm{e}^g \, \frac{x^m}{|\vec{x}|^2} \,. \tag{4.36}$$

Here (t, m) denote world indices in the frame (4.33). For these expressions Maxwell's equations (2.15) are satisfied with the charges defined in (4.28). Observe that, when calculating Maxwell's equations directly from (4.19) in the frame (4.20), one encounters a different sign as compared to (4.28). This is related to the fact that a charge located at the origin in the frame (4.33) corresponds to a charge at infinity in the conformal coordinates used in this section. When evaluating Maxwell's equations in the latter coordinates one is considering the corresponding mirror charge placed at the origin. This explains the apparent sign discrepancy.

5. $N = 1$ supersymmetric field configurations

A general analysis of the conditions for residual $N = 1$ supersymmetry is extremely cumbersome. Therefore we base ourselves on a given class of embeddings of the residual supersymmetry by imposing the following condition on the supersymmetry parameters,

$$h \, \epsilon_i = \varepsilon_{ij} \, \gamma_0 \, \epsilon^j \,, \tag{5.1}$$

where h is some unknown phase factor which is in general not constant, and which transforms under U(1) with the same weight as the fields X^I. At the moment we proceed without imposing gauge choices. Therefore the choice of γ_0 is somewhat arbitrary, because it can be changed into any other gamma matrix by means of a local Lorentz transformation. However, we will eventually impose a gauge condition on the vierbein field, which restricts the local Lorentz transformations to the three-dimensional rotations.[3] It is clear that (5.1) is then consistent with spatial rotations and SU(2) transformations, although we will not require the solutions to be invariant under these symmetries. An embedding condition such as (5.1) was also used in the analysis presented in [3, 4] of $N=2$ theories without R^2-interactions.

Subject to this embedding we can now evaluate the conditions for $N=1$ supersymmetry by following essentially the same steps as in the previous section. We start by considering the variations of the vector multiplet fermions and of the spinors ζ_i^{V} and ζ_i^{H}. They lead to the equations

$$\hat{B}_{ij} = k_{a\,ij} = 0\,, \tag{5.2}$$

and

$$\begin{aligned} \mathcal{A}_0 &= 0\,, \qquad & \mathcal{A}_p &= \mathrm{Re}[h\,\mathcal{F}^-_{0p}]\,,\\ \mathcal{D}_0(\chi \mathrm{e}^{\mathcal{K}}) &= 0\,, \qquad & \mathcal{D}_p(\chi \mathrm{e}^{\mathcal{K}}) &= 2\,\chi \mathrm{e}^{\mathcal{K}}\;\mathrm{Im}[h\,\mathcal{F}^-_{0p}]\,, \end{aligned} \tag{5.3}$$

where the indices $(0,p)$ with $p=1,2,3$ refer to the tangent space. With this result we find that the variation of ζ_i^{V} simplifies considerably and reduces to

$$\delta\zeta_i^{\mathrm{V}} = \chi^{-1}\not{\mathcal{D}}\chi\,\epsilon_i + 2\,\eta_i\,. \tag{5.4}$$

For the hypermultiplets we find the same condition as for full supersymmetry,

$$\mathcal{D}_a(\chi^{-1/2} A_i{}^\alpha) = 0\,. \tag{5.5}$$

Returning to the vector multiplet spinors, we then establish the relations

$$\mathcal{D}_0(\chi^{-1/2}\,X^I) = \mathcal{D}_0(\chi^{-1/2}\,F_I) = 0\,, \tag{5.6}$$

and

$$\begin{aligned} \mathcal{D}_p(\chi^{-1/2}\,X^I) &= -h\,\chi^{-1/2}\left(F^{-I}_{0p} - \frac{1}{4}\varepsilon_{kl}T^{kl}_{0p}\,\bar{X}^I\right),\\ \mathcal{D}_p(\chi^{-1/2}\,F_I) &= -h\,\chi^{-1/2}\left(G^-_{0pI} - \frac{1}{4}\varepsilon_{kl}T^{kl}_{0p}\,\bar{F}_I\right). \end{aligned} \tag{5.7}$$

These last two equations transform covariantly with respect to electric/magnetic duality. Taking their symplectically invariant product with $(\bar{X}^I, \bar{F}_I)$ leads to the previous equations (5.3).

[3]In view of this, Lorentz covariant derivatives should be applied with caution, as the various equations we are about to derive are not Lorentz covariant.

Subsequently we consider the variations of the spinor χ^i, which lead to

$$\begin{aligned} R(\mathcal{V})_{ab}{}^{i}{}_{j} &= 0\,, \\ \mathcal{D}_c(\chi^{1/2}\, T^{ijc0}\,\varepsilon_{ij}) &= -6h\,\chi^{1/2}\,D\,, \\ \mathcal{D}_c(\chi^{1/2}\, T^{ijcp}\,\varepsilon_{ij}) &= 8ih\,\chi^{1/2}\,R(A)^{-0p}\,. \end{aligned} \tag{5.8}$$

Note that the first equation is consistent with the fact that $\hat{B}_{ij}$ vanishes (cf. (2.7)). In view of the fact that the SU(2) field strengths vanish, we will set the SU(2) connections to zero in what follows.

The variations for the field strength $R(Q)^i_{ab}$ lead to

$$\begin{aligned} &\mathcal{D}_0 T^{ij}_{ab} - \frac{1}{2}\chi^{-1}\mathcal{D}_d\chi\left(\delta^d_0\, T^{ij}_{ab} - 2\delta^d_{[a}\, T^{ij}_{b]0} + 2\eta_{0[a}\, T^{ij}_{b]}{}^{d}\right) = 0\,, \\ &\mathcal{D}_p T^{ij}_{ab} - \frac{1}{2}\chi^{-1}\mathcal{D}_d\chi\left(\delta^d_p\, T^{ij}_{ab} - 2\delta^d_{[a}\, T^{ij}_{b]p} + 2\eta_{p[a}\, T^{ij}_{b]}{}^{d}\right) = 4h\,\varepsilon^{ij}\,\mathcal{R}(M)^-_{ab\,0p}\,. \end{aligned} \tag{5.9}$$

Finally we consider the variation of derivatives of fermion fields. The arguments presented below (4.12) about the fact that there is no need to consider more than one of these variations, apply also to residual supersymmetry. Hence we consider the Q-supersymmetry variation of (4.12), making use of the previously obtained results. This yields the following equation,

$$\begin{aligned} &D_\mu(\chi^{-1}D^a\chi) + \frac{1}{4}(\chi^{-1}D_c\chi)^2\, e_\mu{}^a - \frac{1}{2}(\chi^{-1}D_\mu\chi)(\chi^{-1}D^a\chi) = \\ &\quad = -\frac{3}{2}D(e_\mu{}^a - 2e_{\mu0}\,\eta^{a0}) - 2i[R(A)^+ - R(A)^-]_\mu{}^a - 4i\,R(A)^-_{\mu0}\,\eta^{a0}\,. \end{aligned} \tag{5.10}$$

All terms in this equation are real, with the exception of the last term, from which it follows that $R(A)^\pm_{a0}$ must be purely imaginary, so that

$$R(A)_{a0} = \tilde{R}(A)_{pq} = 0\,. \tag{5.11}$$

Just as before, (5.10) fixes the value of the gauge field $f_\mu{}^a$, which takes the (K-invariant) form

$$\begin{aligned} f_\mu{}^a = &-\frac{1}{2}\mathcal{D}_\mu(\chi^{-1}\mathcal{D}^a\chi) - \frac{1}{8}(\chi^{-1}\mathcal{D}_c\chi)^2\, e_\mu{}^a + \frac{1}{4}(\chi^{-1}\mathcal{D}_\mu\chi)(\chi^{-1}\mathcal{D}^a\chi) - \\ &-\frac{3}{4}D(e_\mu{}^a - 2e_{\mu0}\,\eta^{a0}) - i[R(A)^+ - R(A)^-]_\mu{}^a - 2i\,R(A)^-_{\mu0}\,\eta^{a0}\,. \end{aligned} \tag{5.12}$$

Comparing with (2.3) yields

$$\begin{aligned} R(\omega,e)_\mu{}^a - \frac{1}{6}R(\omega,e)\,e^a_\mu = &-\mathcal{D}_\mu(\chi^{-1}\mathcal{D}^a\chi) - \frac{1}{4}(\chi^{-1}\mathcal{D}_c\chi)^2\, e_\mu{}^a + \\ &+\frac{1}{2}(\chi^{-1}\mathcal{D}_\mu\chi)(\chi^{-1}\mathcal{D}^a\chi) - \frac{1}{8}T^{ij}_{\mu b}T^{ab}_{ij} - D(e_\mu{}^a - 3e_{\mu0}\eta^{a0}) + \\ &+i\left[\tilde{R}(A)_{\mu0}\,\eta^{a0} - \tilde{R}(A)_0{}^a\,e_\mu{}^0\right]. \end{aligned} \tag{5.13}$$

Let us briefly return to (5.8) and (5.9) and explore the consequences of (5.11). The first equation of (5.9) yields

$$\mathcal{D}_0 T^{ij\,0p} - \frac{1}{2}\chi^{-1}\mathcal{D}_0\chi\, T^{ij\,0p} + \frac{1}{2}\chi^{-1}\mathcal{D}_q\chi\, T^{ij\,qp} = 0\,. \tag{5.14}$$

Making use of this, the last equation (5.8) leads to

$$\mathcal{D}_q T^{ij\,qp} + \chi^{-1}\mathcal{D}_0\chi\, T^{ij\,0p} = -2ih\,\varepsilon^{ij}\,\tilde{R}(A)^{0p}\,, \tag{5.15}$$

which can be rewritten as

$$\mathcal{D}_{[p}T^{ij}_{q]0}\,\varepsilon_{ij} = 2iR(A)_{pq}h - \frac{1}{2}\chi^{-1}\mathcal{D}_0\chi\, T^{ij}_{pq}\,\varepsilon_{ij}\,. \tag{5.16}$$

Observe that so far we have not imposed any gauge conditions. In order to proceed we will now choose a gauge condition that eliminates the freedom of making (local) scale transformations and conformal boosts. This gauge condition amounts to choosing $b_\mu = 0$ and χ constant. Therefore the covariant derivative $\mathcal{D}_a$ contains only the spin connection fields and the U(1) connection, when appropriate.

In this gauge, (5.16) and the second equation of (5.8) read,

$$\bar{h}\,\mathcal{D}_{[p}T^{ij}_{q]0}\,\varepsilon_{ij} = 2iR(A)_{pq}\,, \qquad \bar{h}\,\mathcal{D}^p T^{ij}_{p0}\,\varepsilon_{ij} = 6\,D\,. \tag{5.17}$$

Furthermore we establish from (5.13) that

$$R(\omega,e) = -3D\,. \tag{5.18}$$

Then, from the second equation of (5.9), one derives the following expressions for the components of the curvature tensor $\mathcal{R}(M)_{ab\,cd}$,

$$\begin{aligned}
\mathcal{R}(M)_{pq\,0r} &= \frac{1}{8}i\varepsilon_{pq}{}^{s}\,\bar{h}\,\mathcal{D}_r T^{ij}_{s0}\,\varepsilon_{ij} + \text{ h.c.}\,,\\
\mathcal{R}(M)_{0r\,pq} &= \frac{1}{8}i\varepsilon_{pq}{}^{s}\,\bar{h}\,\mathcal{D}_s T^{ij}_{r0}\,\varepsilon_{ij} + \text{ h.c.}\,,\\
\mathcal{R}(M)_{0p\,0q} &= -\frac{1}{8}\bar{h}\,\mathcal{D}_q T^{ij}_{p0}\,\varepsilon_{ij} + \text{ h.c.}\,,\\
\mathcal{R}(M)_{pq\,rs} &= \frac{1}{8}\varepsilon_{rs}{}^{v}\varepsilon_{pq}{}^{u}\,\bar{h}\,\mathcal{D}_v T^{ij}_{u0}\,\varepsilon_{ij} + \text{ h.c.}\,.
\end{aligned} \tag{5.19}$$

These expressions satisfy all the constraints (B.5) listed in appendix B, provided one makes use of the relations for $R(A)$ and D (cf. (5.17)). Using (5.12) and the definition of $\mathcal{R}(M)$ allows us to find expressions for the components of the Riemann tensor. Making use of (5.17) we find

$$\begin{aligned}
R(\omega)_{pq\,0r} &= R(\omega)_{0r\,pq}\\
&= \frac{1}{8}\varepsilon_{pq}{}^{s}\left[i\left(\bar{h}\,\mathcal{D}_r T^{ij}_{s0}\,\varepsilon_{ij} - \frac{1}{2}T^{ij}_{r0}\,T_{ij\,s0}\right) + \text{ h.c.}\right],
\end{aligned}$$

$$
\begin{aligned}
R(\omega)_{0p\,0q} &= R(\omega)_{0q\,0p} \\
&= -\frac{1}{8}\left[\left(\bar h\,\mathcal{D}_q T^{ij}_{p0}\,\varepsilon_{ij} + \frac{1}{2}T^{ij}_{q0}\,T_{ij\,p0}\right) + \text{ h.c.}\right] , \\
R(\omega)_{pq\,rs} &= -\frac{1}{2}\delta_{[r[p}\left[\bar h\,\mathcal{D}_{q]}T^{ij}_{s]0}\,\varepsilon_{ij} + \text{ h.c.}\right] + \\
&\quad + \frac{1}{4}\delta_{[r[p}\left[T^{ij}_{q]0}\,T_{ij\,s]0} + T_{ij\,q]0}\,T^{ij}_{s]0} - \delta_{q]s]}\,T^{ij\,v}{}_{0}\,T_{ij\,v0}\right] .
\end{aligned} \tag{5.20}
$$

Here we observe that, owing to (5.17), this result satisfies all the algebraic properties of a Riemann tensor, such as cyclicity and pair exchange. We also note that, by virtue of (5.17) and (5.20) gives rise to (5.13) upon contraction.

At this point we adopt a gauge condition for local Lorentz invariance. We remind the reader that the supersymmetry embedding condition (5.1) is obviously inconsistent with local Lorentz invariance and presupposes that we would eventually impose such a gauge condition. Therefore we bring the vierbein field in block-triangular form by imposing $e_t{}^p = 0$, thereby leaving the SO(3) tangent-space rotations unaffected. Denoting world indices by (t, m), with $m = 1, 2, 3$, we parametrize the vierbein as follows,

$$
e_\mu{}^0 \mathrm{d}x^\mu = \mathrm{e}^g [\, \mathrm{d}t + \sigma_m\, \mathrm{d}x^m \,] , \qquad e_\mu{}^p \mathrm{d}x^\mu = \mathrm{e}^{-g}\, \hat e_m{}^p\, \mathrm{d}x^m , \tag{5.21}
$$

where $\hat e_m{}^p$ is the rescaled dreibein of the three-dimensional space. The corresponding inverse vierbein components are then given by

$$
e_0{}^t = \mathrm{e}^{-g} , \qquad e_0{}^m = 0 , \qquad e_p{}^t = -\sigma_p\, \mathrm{e}^g , \qquad e_p{}^m = \mathrm{e}^g\, \hat e_p{}^m , \tag{5.22}
$$

where, on the right-hand side, spatial tangent-space and world indices are converted by means of the dreibein fields $\hat e_m{}^p$ and its inverse.

Now we concentrate on stationary spacetimes, so that we can adopt coordinates such that the vierbein components are independent of the time coordinate t. In that case the spin connection fields take the following form,

$$
\omega_{l\,pq} = \mathrm{e}^g [\, \hat\omega_{l\,pq} + 2\delta_{l[p}\, \nabla_{q]} g \,] , \qquad \omega_{0\,pq} = \omega_{q\,p0} = -\frac{1}{2}\mathrm{e}^{3g}\, \varepsilon_{pql}\, R(\sigma)^l , \qquad \omega_{0\,0p} = \mathrm{e}^g\, \nabla_p g , \tag{5.23}
$$

where $\hat\omega_m{}^{pq}$ is the spin-connection field associated with the dreibein fields $\hat e$ in the standard way. We used the definition

$$
R(\sigma)^l = \varepsilon^{lpq}\, \nabla_p \sigma_q . \tag{5.24}
$$

Observe that $\nabla^p R(\sigma)_p = 0$. The covariant derivatives ∇_m refer to the three-dimensional space only. Hence they contain the three-dimensional spin connection $\hat\omega_m{}^{pq}$.

The corresponding curvature components take the following form (where we consistently use three-dimensional notation on the right-hand side),

$$
\begin{aligned}
R(\omega)_{pq\,0r} &= \frac{1}{2}\varepsilon_{pq}{}^{s}\,\mathrm{e}^{4g}\Big[\nabla_r R(\sigma)_s + 5\,R(\sigma)_s\,\nabla_r g + R(\sigma)_r\,\nabla_s g - 2\delta_{sr}\,R(\sigma)^u\,\nabla_u g\Big]\,,\\
R(\omega)_{0p\,0q} &= -\mathrm{e}^{2g}\Big[\nabla_p\nabla_q g + 3\,\nabla_p g\,\nabla_q g - \delta_{pq}\,(\nabla_r g)^2\Big] + \\
&\quad + \frac{1}{4}\mathrm{e}^{6g}\Big[R(\sigma)_p\,R(\sigma)_q - \delta_{pq}R(\sigma)^2\Big]\,,\\
R(\omega)_{pq\,rs} &= \mathrm{e}^{2g}\,\hat{R}_{pq\,rs} - 4\,\mathrm{e}^{2g}\,\delta_{[p[r}\left[\nabla_{s]}\nabla_{q]}g + \nabla_{s]}g\,\nabla_{q]}g - \frac{1}{2}\delta_{s]q]}\,(\nabla_u g)^2\right] + \\
&\quad + 3\,\mathrm{e}^{6g}\,\delta_{[p[r}\left[R(\sigma)_{s]}\,R(\sigma)_{q]} - \frac{1}{2}\delta_{s]q]}\,R(\sigma)^2\right]\,.
\end{aligned}
\tag{5.25}
$$

However, for stationary solutions also other quantities than those that encode the spacetime should be time-independent. Hence we infer that $\bar{h}X^I$, $\bar{h}F_I$, and $\bar{h}T^{ij}_{p0}$ are time-independent while $(\partial_t + iA_t)h = 0$.

Until now we have restricted our attention to quantities that are supercovariant with respect to full $N=2$ supersymmetry. However, when considering residual supersymmetry, certain linear combinations of the gravitini will still transform covariantly. To see how this works, let us record the gravitini transformation rules in the restricted background. Here we make use of (5.4) to argue that there is no need for including compensating S-supersymmetry transformations. The result takes the form

$$
\begin{aligned}
\delta\psi^i_t &= 2\partial_t\epsilon^i + iA_t\,\epsilon^i + \mathrm{e}^{2g}\left[T_p - \nabla_p g + \frac{1}{2}i\mathrm{e}^{2g}\,R(\sigma)_p\right]\,\gamma^p\gamma_0\epsilon^i\,,\\
\delta\psi^i_m &= 2\nabla_m\epsilon^i - (T_m - iA_m)\epsilon^i - i\hat{e}_m^{\;p}\,\varepsilon_p{}^{qr}\left[T_r - \nabla_r g + \frac{1}{2}i\mathrm{e}^{2g}\,R(\sigma)_r\right]\,\gamma_q\gamma_0\epsilon^i + \\
&\quad + \sigma_m\,\mathrm{e}^{2g}\left[T_p - \nabla_p g + \frac{1}{2}i\mathrm{e}^{2g}\,R(\sigma)_p\right]\,\gamma^p\gamma_0\epsilon^i\,,
\end{aligned}
\tag{5.26}
$$

where we have introduced a three-dimensional world vector T_m,

$$
T_m \equiv \frac{1}{4}\mathrm{e}^{-g}\,\hat{e}_m{}^p\,\bar{h}T^{ij}_{p0}\,\varepsilon_{ij}\,. \tag{5.27}
$$

Now we observe that the combinations $\psi_{\mu i} - \bar{h}\,\varepsilon_{ij}\gamma_0\psi^j_\mu$ transform covariantly under the residual supersymmetry. From the requirement that these covariant variations vanish we deduce directly that

$$
T_m = \nabla_m g - \frac{1}{2}i\mathrm{e}^{2g}\,R(\sigma)_m\,, \qquad \bar{h}\nabla_m h + iA_m = -\frac{1}{2}i\mathrm{e}^{2g}\,R(\sigma)_m\,. \tag{5.28}
$$

This leads to the following expressions for the gravitini variations,

$$
\delta\psi^i_t = 2\partial_t\epsilon^i + iA_t\epsilon^i\,, \qquad \delta\psi^i_m = 2\nabla_m\epsilon^i - (\nabla_m g + \bar{h}\nabla_m h)\epsilon^i\,. \tag{5.29}
$$

With these results we return to the previous identities and verify whether they are now satisfied. It is straightforward to see that this is the case for (5.14). For the other identities one needs the covariant derivative $\bar{h}\,\mathcal{D}_p T^{ij}_{q0}$, which, in three-dimensional notation, reads

$$\bar{h}\,\mathcal{D}_p T^{ij}_{q0}\varepsilon_{ij} = 4\,\mathrm{e}^{2g}\left[\nabla_p T_q + 2\,T_p T_q - \delta_{pq}\,(T_r)^2\right]. \tag{5.30}$$

It is now straightforward to prove (5.17) with D given by

$$D = \frac{2}{3}\mathrm{e}^{2g}\left[\nabla_p^2 g - (\nabla_p g)^2 + \frac{1}{4}\mathrm{e}^{4g}\,(R(\sigma)_p)^2\right]. \tag{5.31}$$

Furthermore, it turns out that (5.20) and (5.25) agree, provided that the curvature of the three-space is zero,

$$\hat{R}_{mn\,pq} = 0\,, \tag{5.32}$$

so that the three-dimensional space is flat. Observe that this result is consistent with the integrability condition corresponding to the Killing spinor equations that one obtains when setting the gravitino variations (5.29) to zero. The only remaining equations are now (5.7), which express the abelian field strengths in terms of the other fields,

$$\begin{aligned} F^{-I}_{0p} &= -\mathrm{e}^{g}\left[\nabla_p(\bar{h}X^I) + (\nabla_p g)\,h\bar{X}^I - \frac{1}{2}i\mathrm{e}^{2g}R(\sigma)_p(\bar{h}X^I + h\bar{X}^I)\right], \\ G^{-}_{0pI} &= -\mathrm{e}^{g}\left[\nabla_p(\bar{h}F_I) + (\nabla_p g)\,h\bar{F}_I - \frac{1}{2}i\mathrm{e}^{2g}R(\sigma)_p(\bar{h}F_I + h\bar{F}_I)\right], \end{aligned} \tag{5.33}$$

where on the right-hand side, we consistently use three-dimensional tangent space indices. With these results we derive the following expressions,

$$\begin{aligned} \mathcal{D}^a F^{-I}_{ap} &= i\mathrm{e}^{g}\,\varepsilon_p{}^{qr}\nabla_q F^{-I}_{0r} \\ &= -\frac{1}{2}\mathrm{e}^{g}\,\varepsilon_p{}^{qr}\nabla_q\left[\mathrm{e}^{3g}R(\sigma)_r(\bar{h}X^I + h\bar{X}^I)\right] - i\mathrm{e}^{2g}\,\varepsilon_p{}^{qr}\,\nabla_q g\,\nabla_r(\bar{h}X^I - h\bar{X}^I)\,, \\ \mathcal{D}^a F^{-I}_{a0} &= \mathrm{e}^{g}\left[\nabla^q F^{-I}_{q0} - 2\left(\nabla^q g - \frac{1}{2}i\mathrm{e}^{2g}R(\sigma)^q\right)F^{-I}_{q0}\right] \\ &= \mathrm{e}^{2g}\left[\nabla_p^2(\bar{h}X^I) + (\nabla_p^2 g)\,h\bar{X}^I - (\nabla_p g)^2\,h\bar{X}^I + (\nabla_p g)\,\nabla^p(h\bar{X}^I - \bar{h}X^I) - \right. \\ &\qquad \left. -\frac{1}{2}i\mathrm{e}^{3g}R(\sigma)^p\,\nabla_p\left[\mathrm{e}^{-g}(h\bar{X}^I - \bar{h}X^I)\right] + \frac{1}{2}\mathrm{e}^{4g}(R(\sigma)_p)^2(h\bar{X}^I + \bar{h}X^I)\right], \end{aligned} \tag{5.34}$$

and likewise for the electric/magnetic dual equations (i.e. replacing F^{-I} by G^-_I, etc.). The imaginary parts of the above expressions correspond to Maxwell's equations for the abelian vector fields. Because the first expression is manifestly real, the corresponding Maxwell equation (and its electric/magnetic dual) is satisfied. The imaginary part of the second expression and its dual equation provide the remaining Maxwell equations, which read

$$\begin{aligned}\nabla_p^2\Big[\mathrm{e}^{-g}(\bar hX^I-h\bar X^I)\Big]&=0\,,\\ \nabla_p^2\Big[\mathrm{e}^{-g}(\bar hF_I-h\bar F_I)\Big]&=0\,,\end{aligned}\tag{5.35}$$

which shows that the functions in parentheses are harmonic. Furthermore we note the equations

$$\begin{aligned}F_{0p}^{-I}+F_{0p}^{+I}&=-\nabla_p\Big[\mathrm{e}^{g}(\bar hX^I+h\bar X^I)\Big]\,,\\ G_{0pI}^{-}+G_{0pI}^{+}&=-\nabla_p\Big[\mathrm{e}^{g}(\bar hF_I+h\bar F_I)\Big]\,,\end{aligned}\tag{5.36}$$

so that the functions under the derivative can be regarded as electric and magnetic potentials.

So far our analysis is valid for any chiral background. Now we identify this background with (2.7) and note that the field $\hat A$ can be written as

$$\hat A=-64\mathrm{e}^{2g}\,h^2\,(T_p)^2\,.\tag{5.37}$$

With this choice for the background we now evaluate the field equations for the fields D and A_μ, which were given in (4.16) and (4.17), respectively. Using (5.7) and (5.31) and the homogeneity properties of $F(X,\hat A)$, the first equation takes the form

$$\begin{aligned}\mathrm{e}^{-\mathcal K}+\frac12\chi=&-128i\,\mathrm{e}^{3g}\,\nabla^p\Big[\mathrm{e}^{-g}\,\nabla_pg\,(F_A-\bar F_A)\Big]-32i\,\mathrm{e}^{6g}\,(R(\sigma)_p)^2(F_A-\bar F_A)-\\ &-64\,\mathrm{e}^{4g}R(\sigma)_p\,\nabla^p(F_A+\bar F_A)\,.\end{aligned}\tag{5.38}$$

The second equation (4.17) comprises four equations. The one with $a=0$ turns out to be identically satisfied, by virtue of of an intricate interplay of all the results that we obtained above. This constitutes a very subtle check upon the correctness of the results obtained so far. Using similar manipulations equation (4.17) with $a=p$ can be written as

$$\begin{aligned}&(\bar hX^I-h\bar X^I)\overset{\leftrightarrow}{\nabla}_p(\bar hF_I-h\bar F_I)-\frac12\chi\,\mathrm{e}^{2g}\,R(\sigma)_p=\\ &\quad=128\,\mathrm{e}^{2g}\,\nabla^q\Big[2\nabla_{[p}g\,\nabla_{q]}(F_A+\bar F_A)+i\nabla_{[p}\Big(\mathrm{e}^{2g}\,R(\sigma)_{q]}\,(F_A-\bar F_A)\Big)\Big]\,.\end{aligned}\tag{5.39}$$

To arrive at this concise expression requires an extensive usage of many of the previously obtained results, and in particular of (5.38).

This concludes our analysis. The solutions can now be expressed in terms of harmonic functions according to (5.35). The two field equations (5.38) and (5.39) then determine the function g and $R(\sigma)_p$, from which all other quantities of interest follow. We should point out that there are some equations of motion whose validity has not yet been verified. We claim that those are implied by the residual supersymmetry of our solutions. For instance, for the vector multiplets we have imposed the Maxwell equations. Therefore the $N = 1$ supersymmetry variation of the field equations of the vector multiplet fermions can only lead to the field equations of the vector multiplet scalars, which must thus be satisfied by supersymmetry. For the hypermultiplets a similar argument holds. Indeed, the result (5.18), which is crucial for the validity of the field equation for the hypermultiplet scalars, has already been established on the basis of the previous analysis. The field equations for the fields of the Weyl multiplet have been imposed, with the exception of those for the vierbein field and the tensor field T_{ijab} (the field equations for the SU(2) gauge fields are trivially satisfied because of the SU(2) symmetry of our solutions). However, the field equations of the gravitino fields and of the fermion doublet χ^i transform into these two field equations, from which one may conclude that they are also satisfied by supersymmetry.

6. Discussion and conclusions

In this paper we have characterized all stationary solutions with a residual $N = 1$ supersymmetry embedded according to (5.1). In principle there may exist other solutions based on inequivalent embeddings of $N = 1$ supersymmetry. It should be interesting to apply our approach to more general embeddings of the residual supersymmetry.

By imposing the conditions for residual supersymmetry and a subset of the field equations we have obtained the full class of these solutions, albeit not explicitly because the equations depend on the holomorphic function $F(X, \hat{A})$ that characterizes both the vector multiplets and the R^2-interactions. A gratifying feature of our results is that the presence of the R^2-interactions gives rise to relatively minor complications, something that may seem rather surprising in view of the complicated structure of the higher-derivative terms in the action. There are two reasons for the fact that these complications can remain so implicit in our analysis. The first is that the higher-derivative interactions are nicely encoded in the holomorphic function $F(X, \hat{A})$. The second reason is that we have consistently used quantities that transform covariantly under electric/magnetic duality. Without this guidance there would be a multitude of ways to express our results and perform the analysis.

We have also shown that solutions with supersymmetry enhancement exhibit fixed-point behavior of the moduli fields, simply because the solutions with full $N = 2$ supersymmetry are unique. This result is relevant when calculating the horizon geometry of extremal black holes since it explains why the black hole entropy depends only on the electric and magnetic charges carried by the black hole.

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Let us briefly summarize the solutions that we have found. Following [2] we introduce the rescaled U(1) and Weyl invariant variables,

$$Y^I = \mathrm{e}^{-g}\,\bar{h}\,X^I\,, \qquad \Upsilon = \mathrm{e}^{-2g}\,\bar{h}^2\,\hat{A}\,. \tag{6.1}$$

Using the homogeneity of $F(X,\hat{A})$, we can write $F(Y,\Upsilon) = \exp[-2g]\,\bar{h}^2 F(X,\hat{A})$ and

$$\begin{pmatrix} Y^I \\ F_I(Y,\Upsilon) \end{pmatrix} = \mathrm{e}^{-g}\,\bar{h}\begin{pmatrix} X^I \\ F_I(X,\hat{A}) \end{pmatrix}. \tag{6.2}$$

Observe that $F_A(X,\hat{A}) = F_\Upsilon(Y,\Upsilon)$. Henceforth we will always use the rescaled variables. The rescaled background field Υ is given by

$$\Upsilon = -64\left(\nabla_m g - \frac{1}{2}i\mathrm{e}^{2g}\,R(\sigma)_m\right)^2. \tag{6.3}$$

Furthermore from (3.6) and (6.1) we infer that

$$\mathrm{e}^{-2g} = i\,\mathrm{e}^{\mathcal{K}}\left[\bar{Y}^I F_I(Y,\Upsilon) - \bar{F}_I(\bar{Y},\bar{\Upsilon})Y^I\right]. \tag{6.4}$$

According to (5.35) we can express the imaginary part of (Y^I, F_J) in terms of a symplectic array of $2(n+1)$ harmonic functions $(H^I(\vec{x}), H_J(\vec{x}))$,

$$\begin{pmatrix} Y^I - \bar{Y}^I \\ F_I(Y,\Upsilon) - \bar{F}_I(\bar{Y},\bar{\Upsilon}) \end{pmatrix} = i\begin{pmatrix} H^I \\ H_I \end{pmatrix}. \tag{6.5}$$

These are the "generalized stabilization equations" which were conjectured in [3] and [5] for the case without and with R^2-interactionsrespectively (a derivation for certain solutions without R^2-terms appeared in [11]). In principle these equations determine the full spatial dependence of Y^I in terms of the harmonic functions and the background field Υ. However, explicit solutions of the stabilization equations can only be obtained in a small number of cases and usually one has to solve the equations by iteration which is extremely cumbersome. We discuss a few examples of explicit solutions in [18].

We write the harmonic functions as a linear combination of several harmonic functions associated with multiple centers located at $\vec{x}_A$ with electric charges q_{AI} and magnetic charges p^I_A,

$$H^I(\vec{x}) = h^I + \sum_A \frac{p^I_A}{|\vec{x}-\vec{x}_A|}\,, \qquad H_I(\vec{x}) = h_I + \sum_A \frac{q_{AI}}{|\vec{x}-\vec{x}_A|}\,, \tag{6.6}$$

where the (h^I, h_J) are constants and the charges are normalized according to (2.15). Furthermore, we recall

$$\begin{aligned} F^{-I}_{0p} &= -\mathrm{e}^{2g}\left[\nabla_p Y^I + \left(\nabla_p g - \frac{1}{2}i\mathrm{e}^{2g}R(\sigma)_p\right)(Y^I + \bar{Y}^I)\right], \\ G^{-}_{0pI} &= -\mathrm{e}^{2g}\left[\nabla_p F_I + \left(\nabla_p g - \frac{1}{2}i\mathrm{e}^{2g}R(\sigma)_p\right)(F_I + \bar{F}_I)\right], \end{aligned} \tag{6.7}$$

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and hence

$$F_{0p}^{-I} + F_{0p}^{+I} = -\nabla_p \Big[e^{2g} (Y^I + \bar{Y}^I) \Big] \,,$$
$$G_{0pI}^{-} + G_{0pI}^{+} = -\nabla_p \Big[e^{2g} (F_I + \bar{F}_I) \Big] \,. \tag{6.8}$$

We also rewrite the expressions (5.38) and (5.39) in terms of the rescaled variables,

$$\begin{aligned} e^{-\mathcal{K}} + \frac{1}{2}\chi &= -128 i\, e^{3g}\, \nabla^p \Big[e^{-g}\, \nabla_p g\, (F_\Upsilon - \bar{F}_\Upsilon) \Big] - 32 i\, e^{6g}\, (R(\sigma)_p)^2 (F_\Upsilon - \bar{F}_\Upsilon) - \\ &\quad - 64\, e^{4g} R(\sigma)_p\, \nabla^p (F_\Upsilon + \bar{F}_\Upsilon) \,, \end{aligned} \tag{6.9}$$

$$\begin{aligned} H^I \overset{\leftrightarrow}{\nabla}_p H_I &= -\frac{1}{2}\chi\, R(\sigma)_p - \\ &\quad - 128 \nabla^q \Big[2 \nabla_{[p} g\, \nabla_{q]} (F_\Upsilon + \bar{F}_\Upsilon) + i \nabla_{[p} \Big(e^{2g} R(\sigma)_{q]}\, (F_\Upsilon - \bar{F}_\Upsilon) \Big) \Big] \,. \end{aligned} \tag{6.10}$$

We note that both sides of (6.10) are manifestly divergence free away from the centers. Furthermore, in the one-center case where the solution has spherical symmetry and depends only on the radial coordinate, the terms involving F_Υ and its complex conjugate vanish in (6.10).

Let us first briefly discuss the solutions in the absence of R^2-interactions. Then (6.9) and (6.10) imply that

$$e^{-\mathcal{K}} + \frac{1}{2}\chi = 0 \,, \qquad R(\sigma)_m = -2\, \chi^{-1}\, H^I \overset{\leftrightarrow}{\nabla}_m H_I \,. \tag{6.11}$$

Once we have solved the stabilization equations, we have thus constructed the full solution in terms of the harmonic functions. For the static solutions, where $R(\sigma)_m = 0$, this implies that $H^I \overset{\leftrightarrow}{\nabla}_m H_I = 0$, which leads to the following condition on the charges [3],

$$h^I\, q_{AI} - h_I\, p_A^I = 0 \,, \qquad p_A^I\, q_{BI} - q_{AI}\, p_B^I = 0 \,. \tag{6.12}$$

The second condition implies that the charges are mutually local, i.e. the solution can be related to one carrying electric charges only by electric/magnetic duality. Moreover it implies that the total angular momentum of a dyon A in the field of a dyon B vanishes.

Asymptotically, at spatial infinity, the fields can be expanded in powers of $1/|\vec{x}|$,

$$Y^I(\vec{x}) = Y^I(\infty) + \frac{y^I}{|\vec{x}|} + \cdots \,, \qquad F_I(\vec{x}) = F_I(\infty) + \frac{f_I}{|\vec{x}|} + \cdots \,. \tag{6.13}$$

Inspection of (6.5) then shows that $Y^I(\infty) - \bar{Y}^I(\infty) = i h^I$, $F_I(\infty) - \bar{F}_I(\infty) = i h_I$ as well as $y^I - \bar{y}^I = i p^I$ and $f_I - \bar{f}_I = i q_I$, where p^I and q_I denote the (total) magnetic and electric charges, respectively. The homogeneity of the holomorphic function F implies $F_I\, \delta Y^I - Y^I\, \delta F_I = 0$, and therefore we conclude that $y^I F_I(\infty) - f_I Y^I(\infty) = 0$.

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The following results can then be obtained by explicit calculation,

$$\begin{aligned}\vec{R}(\sigma)(\vec{x}) &= \mathrm{e}^{\mathcal{K}}\Big[h_I p^I - h^I q_I\Big]\,\frac{\vec{x}}{|\vec{x}|^3} + \cdots\,,\\ \mathrm{e}^{-\mathcal{K}-2g} &= \Big[\mathrm{e}^{-\mathcal{K}-2g}\Big]_\infty\,\Big\{1+\Big[\mathrm{e}^{\mathcal{K}/2+g}\Big]_\infty\,\frac{2M_{\rm ADM}}{|\vec{x}|}+\cdots\Big\}\,,\end{aligned}\tag{6.14}$$

where $M_{\rm ADM}$ denotes the ADM mass (in Planck units),

$$M_{\rm ADM} = \frac{1}{2}\Big[\mathrm{e}^{\mathcal{K}/2+g}\Big]_\infty\,\Big(p^I F_I(\infty) - q_I Y^I(\infty) + \text{h.c.}\Big)\,. \tag{6.15}$$

Note that the $M_{\rm ADM}$ can be written as $M_{\rm ADM} = \frac{1}{2}[\bar{h}Z(\infty)+h\bar{Z}(\infty)]$, where Z was defined in (4.29). For static solutions $\bar{h}Z$ is real by virtue of the first condition in (6.12), so that $M_{\rm ADM} = \bar{h}Z(\infty)$ [3]. With these results one easily shows that the electric and magnetic fields (6.7) have the characteristic $1/|\vec{x}|^2$ fall-off at spatial infinity.

We now discuss the solutions with R^2-interactions. In the presence of these interactions the equations (6.9) and (6.10) are more difficult to analyze. We note that, generically, multi-centered solutions satisfying $H^I \overset{\leftrightarrow}{\nabla}_p H_I = 0$ are not static, since (6.10) then reads

$$R(\sigma)_p = -256\,\chi^{-1}\,\nabla^q\Big[2\nabla_{[p}g\,\nabla_{q]}(F_\Upsilon+\bar{F}_\Upsilon) + i\nabla_{[p}\Big(\mathrm{e}^{2g}\,R(\sigma)_{q]}\,(F_\Upsilon-\bar{F}_\Upsilon)\Big)\Big]\,. \tag{6.16}$$

Examples of black holes exhibiting this feature are discussed in [18].

When a solution has a horizon with full supersymmetry, we can connect the results of this section to those of section 4. In doing so, it is important to keep in mind that we used a different parametrization of the metric in section 4 (cf. (4.20)). The results can be connected through the following identifications, which are valid at the horizon (which we take to be located at $|\vec{x}|=0$, for convenience),

$$\begin{aligned}&Y^I \approx \frac{[\mathrm{e}^{\mathcal{K}/2}\bar{Z}X^I]_{\rm hor}}{|\vec{x}|}\,, &&F_I(Y)\approx\frac{[\mathrm{e}^{\mathcal{K}/2}\bar{Z}F_I(X)]_{\rm hor}}{|\vec{x}|}\,,\\ &\mathrm{e}^{-g}\approx\frac{[\mathrm{e}^{\mathcal{K}/2}|Z|]_{\rm hor}}{|\vec{x}|}\,, &&h\approx\frac{Z}{|Z|}\Big|_{\rm hor}\,, \qquad \Upsilon\approx-\frac{64}{|\vec{x}|^2}\,.\end{aligned}\tag{6.17}$$

In particular, when approaching the horizon, the expressions for the field strengths (6.7) coincide with (4.36).

In the presence of R^2-interactions, the homogeneity of the holomorphic function $F(Y,\Upsilon)$ implies $F_I\,\delta Y^I - Y^I\,\delta F_I = 2\Upsilon\,\delta F_\Upsilon$. If we assume that, at spatial infinity, the fields Y^I, F_I and e^{-g} have an asymptotic expansion of the type (6.13), and if we furthermore assume that $\Upsilon\,\delta F_\Upsilon$ falls off to zero sufficiently rapidly so that we have $y^I F_I(\infty) - f_I Y^I(\infty) = 0$, then the ADM mass of the solution is still given by (6.15).

Finally, let us point out that the multi-centered solutions we have studied can now be used as a starting point for computing the metric on the moduli space of four-dimensional extremal black holes in the presence of R^2-interactions. In the absence of R^2-interactions, it was found [16, 17] that the metric on the moduli space of electrically charged BPS black holes is determined in terms of a moduli potential μ given by $\mu = -\frac{1}{2}\chi \int d^3\vec{x}\, \mathrm{e}^{-4g}$. As shown above, when turning on R^2-interactions, e^{-2g} itself gets modified according to (6.4) with $\mathrm{e}^{\mathcal{K}}$ given by (6.9). Thus, the moduli potential receives R^2-corrections which are encoded in e^{-4g} (and possibly further corrections due to additional explicit modifications of μ). Using (6.9) we can rewrite μ as follows,

$$
\begin{aligned}
\mu &= -\frac{1}{2}\chi \int d^3x\ \mathrm{e}^{-4g} \\
&= \int d^3x\ \mathrm{e}^{-4g}\Big(\mathrm{e}^{-\mathcal{K}} + 256\,\mathrm{e}^{3g}\,\nabla_m\Big[\mathrm{Im}\ F_\Upsilon\ \nabla^m \mathrm{e}^{-g}\Big] - \\
&\qquad\qquad - \ 64\,\mathrm{e}^{6g}\,(R(\sigma)_m)^2\ \mathrm{Im}\ F_\Upsilon + 128\,\mathrm{e}^{4g}\,R(\sigma)^m\,\nabla_m\ \mathrm{Re}\ F_\Upsilon\Big) \\
&= \int d^3x\ \mathrm{e}^{-2g}\Big(i\Big[\bar{Y}^I F_I(Y,\Upsilon) - \bar{F}_I(\bar{Y},\bar{\Upsilon})Y^I\Big] + 4\,|\Upsilon|\ \mathrm{Im}\ F_\Upsilon\Big)\,.
\end{aligned}
\tag{6.18}
$$

Observe that the combination $i[\bar{Y}^I F_I(Y,\Upsilon) - \bar{F}_I(\bar{Y},\bar{\Upsilon})Y^I] + 4\,|\Upsilon|\ \mathrm{Im}\ F_\Upsilon$, when evaluated at the horizon of a BPS black hole, is precisely equal to $\pi^{-1}|\vec{x}|^{-2}$ times the expression for its macroscopic entropy (see (4.32) and (6.17))! This intriguing feature of (6.18) may indicate that there are no additional explicit modifications of μ due to R^2-interactions. This is currently under investigation [19]. In establishing (6.18) we dropped certain boundary terms when integrating by parts. Some of those are known to be proportional to $|\vec{x}_A - \vec{x}_B|^{-1}$ (for two non-coincident centers A and B) and therefore do not contribute to the metric on the moduli space [16].

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A. Notation and conventions

We denote spacetime indices by $\mu, \nu, \ldots$, and Lorentz indices by $a, b, \ldots = 0, 1, 2, 3$. Indices $i, j, k, \ldots$ are usually reserved for SU(2)-indices. Our (anti)symmetrization conventions are

$$
[ab] = \frac{1}{2}(ab - ba)\,, \qquad (ab) = \frac{1}{2}(ab + ba)\,. \tag{A.1}
$$

We take

$$\gamma_a\gamma_b = \eta_{ab} + \gamma_{ab}\,, \qquad \gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3\,, \tag{A.2}$$

where η_{ab} is of signature $(-+++)$. The complete antisymmetric tensor satisfies

$$\varepsilon^{abcd} = e^{-1}\varepsilon^{\mu\nu\lambda\sigma}e_\mu^a e_\nu^b e_\lambda^c e_\sigma^d\,, \qquad \varepsilon^{0123} = i\,, \tag{A.3}$$

which implies

$$\gamma_{ab} = -\frac{1}{2}\varepsilon_{abcd}\gamma^{cd}\gamma_5\,. \tag{A.4}$$

The dual of an antisymmetric tensor field F_{ab} is given by

$$\tilde{F}_{ab} = \frac{1}{2}\varepsilon_{abcd}F^{cd}\,, \tag{A.5}$$

and the (anti-)selfdual part of F_{ab} reads

$$F^{\pm}_{ab} = \frac{1}{2}(F_{ab} \pm \tilde{F}_{ab})\,. \tag{A.6}$$

We note the following useful identities for (anti-)selfdual tensors in 4 dimensions:

$$\begin{aligned}
G^{\pm}_{[a[c}\,H^{\pm}_{d]b]} &= \pm\frac{1}{8}G^{\pm}_{ef}\,H^{\pm ef}\,\varepsilon_{abcd} - \frac{1}{4}(G^{\pm}_{ab}\,H^{\pm}_{cd} + G^{\pm}_{cd}\,H^{\pm}_{ab})\,,\\
G^{\pm}_{ab}\,H^{\mp cd} + G^{\pm cd}\,H^{\mp}_{ab} &= 4\delta_{[a}^{[c}G^{\pm}_{b]e}\,H^{\mp d]e}\,,\\
\frac{1}{2}\varepsilon^{abcd}\,G^{\pm e}_{[c}\,H^{\pm}_{d]e} &= \pm G^{\pm[a}{}_{e}\,H^{\pm b]e}\,,\\
G^{\pm ac}\,H^{\pm b}_{c} + G^{\pm bc}\,H^{\pm a}_{c} &= -\frac{1}{2}\eta^{ab}\,G^{\pm cd}\,H^{\pm}_{cd}\,,\\
G^{\pm ac}\,H^{\mp b}_{c} &= G^{\pm bc}\,H^{\mp a}_{c}\,, \qquad G^{\pm ab}\,H^{\mp}_{ab} = 0\,.
\end{aligned} \tag{A.7}$$

Note that under hermitean conjugation (h.c.) selfdual becomes anti-selfdual and vice-versa. Any SU(2) index i or any quaternionic index α changes position under h.c., for instance

$$(T_{ab\,ij})^* = T_{ab}^{ij}\,, \qquad (A_i^{\alpha})^* = A_\alpha^i\,. \tag{A.8}$$

B. Superconformal calculus

The superconformal algebra consists of general coordinate, local Lorentz, dilatation, special conformal, chiral U(1) and SU(2), and Q- and S-supersymmetry transformations. The fully supercovariant derivatives are denoted by D_a. We use $\mathcal{D}_\mu$ to denote a covariant derivative with respect to Lorentz, dilatation, chiral $\mathrm{U}(1), \mathrm{SU}(2)$ and gauge transformations. The component fields of the various superconformal multiplets carry certain Weyl and chiral weights. Those of the Weyl multiplet and of the supersymmetry transformation parameters are listed in table 1, whereas those of the vector and of hypermultiplets are given in table 2. These tables also list the fermion

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	Weyl multiplet											parameters	
field	$e_\mu{}^a$	ψ_μ^i	b_μ	A_μ	$\mathcal{V}_\mu{}^i{}_j$	T_{ab}^{ij}	χ^i	D	ω_μ^{ab}	$f_\mu{}^a$	ϕ_μ^i	ϵ^i	η^i
w	-1	$-\frac{1}{2}$	0	0	0	1	$\frac{3}{2}$	2	0	1	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$
c	0	$-\frac{1}{2}$	0	0	0	-1	$-\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$
γ_5		+					+				$-$	+	$-$

Table 1: Weyl and chiral weights (w and c, respectively) and fermion chirality (γ_5) of the Weyl multiplet component fields and of the supersymmetry transformation parameters.

	vector multiplet				hypermultiplet	
field	X^I	Ω_i^I	W_μ^I	Y_{ij}^I	A_i^α	ζ^α
w	1	$\frac{3}{2}$	0	2	1	$\frac{3}{2}$
c	-1	$-\frac{1}{2}$	0	0	0	$-\frac{1}{2}$
γ_5		+				$-$

Table 2: Weyl and chiral weights (w and c, respectively) and fermion chirality (γ_5) of the vector and hypermultiplet component fields.

chirality of the various component fields. To exhibit the form of the derivatives $\mathcal{D}_\mu$ and the normalization of the gauge fields contained in them, let us give the derivative of the chiral spinor ϵ^i,

$$\mathcal{D}_\mu \epsilon^i = \partial_\mu \epsilon^i - \frac{1}{4}\omega_\mu^{ab}\,\gamma_{ab}\epsilon^i + \frac{1}{2}(b_\mu + iA_\mu)\epsilon^i + \frac{1}{2}\mathcal{V}_\mu{}^i{}_j\epsilon^j \,. \tag{B.1}$$

The gauge fields for Lorentz and S-supersymmetry transformations are composite objects and given by

$$\begin{aligned} \omega_\mu^{ab} &= -2e^{\nu[a}\partial_{[\mu}e_{\nu]}{}^{b]} - e^{\nu[a}e^{b]\sigma}e_{\mu c}\partial_\sigma e_\nu{}^c - 2e_\mu{}^{[a}e^{b]\nu}b_\nu - \\ &\quad - \frac{1}{4}\left(2\bar\psi_\mu^i\gamma^{[a}\psi_i^{b]} + \bar\psi^{ai}\gamma_\mu\psi_i^b + \text{h.c.}\right) , \\ \phi_\mu^i &= \frac{1}{2}\left(\gamma^{\rho\sigma}\gamma_\mu - \frac{1}{3}\gamma_\mu\gamma^{\rho\sigma}\right)\left(\mathcal{D}_\rho\psi_\sigma^i - \frac{1}{16}T^{ab\,ij}\gamma_{ab}\gamma_\rho\psi_{\sigma j} + \frac{1}{4}\gamma_{\rho\sigma}\chi^i\right) \\ &= -\frac{1}{3}\left(4\delta_\mu^{[\rho}\gamma^{\sigma]} + \varepsilon_{\mu\lambda}{}^{\rho\sigma}\gamma^\lambda\right)\left(\mathcal{D}_\rho\psi_\sigma^i - \frac{1}{16}T^{ab\,ij}\gamma_{ab}\gamma_\rho\psi_{\sigma j} + \frac{1}{4}\gamma_{\rho\sigma}\chi^i\right), \end{aligned} \tag{B.2}$$

respectively. The gauge field for special conformal transformations is also a composite object and was already given in (2.3), up to fermionic terms. There is no need to give the transformation rules for the dependent gauge fields. The explicit transformation rule for ϕ_μ^i is, however, used in the calculations of this paper and we have presented it in (2.2). Throughout this paper we need certain supercovariant curvature tensors,

$$R(Q)_{\mu\nu}^i = 2\mathcal{D}_{[\mu}\psi_{\nu]}^i - \gamma_{[\mu}\phi_{\nu]}^i - \frac{1}{8}T^{ab\,ij}\gamma_{ab}\gamma_{[\mu}\psi_{\nu]j} \,,$$

$$
\begin{aligned}
R(A)_{\mu\nu} &= 2\partial_{[\mu}A_{\nu]} - i\left(\frac{1}{2}\bar{\psi}^i_{[\mu}\phi_{\nu]i} + \frac{3}{4}\bar{\psi}^i_{[\mu}\gamma_{\nu]}\chi_i - \text{h.c.}\right), \\
R(\mathcal{V})_{\mu\nu}{}^i{}_j &= 2\partial_{[\mu}\mathcal{V}^i_{\nu]j} + \mathcal{V}^i_{[\mu k}\mathcal{V}^k_{\nu]j} + \left(2\bar{\psi}^i_{[\mu}\phi_{\nu]j} - 3\bar{\psi}^i_{[\mu}\gamma_{\nu]}\chi_j - (\text{h.c.}\,;\ \text{traceless})\right), \\
R(M)^{ab}_{\mu\nu} &= R(\omega)^{ab}_{\mu\nu} - 4f_{[\mu}^{[a}e_{\nu]}^{b]} + \left(\frac{1}{2}\bar{\psi}^i_{[\mu}\gamma^{ab}\phi_{\nu]i} + \text{h.c.}\right) + \\
&\quad + \left(\frac{1}{2}\bar{\psi}^i_{[\mu}T^{ab}_{ij}\phi^i_{\nu]} - \frac{3}{4}\bar{\psi}^i_{[\mu}\gamma_{\nu]}\gamma^{ab}\chi^i - \bar{\psi}^i_{[\mu}\gamma_{\nu]}R_{\mu\nu}(Q)_i + \text{h.c.}\right), \\
R(S)^i_{\mu\nu} &= 2\mathcal{D}_{[\mu}\phi^i_{\nu]} - 2f^a_{[\mu}\gamma_a\psi^i_{\nu]} - \frac{1}{8}\not{D}T^{ij}_{ab}\gamma^{ab}\gamma_{[\mu}\psi_{\nu]j} - 3\bar{\chi}_j\gamma^a\psi^j_{[\mu}\gamma_a\psi^i_{\nu]} + \\
&\quad + \frac{1}{4}R(\mathcal{V})_{ab}{}^i{}_j\gamma^{ab}\gamma_{[\mu}\psi^j_{\nu]} + \frac{1}{2}iR(A)_{ab}\gamma^{ab}\gamma_{[\mu}\psi^i_{\nu]}, \\
R(D)_{\mu\nu} &= 2\partial_{[\mu}b_{\nu]} - 2f^a_{[\mu}\,e_{\nu]a} - \left(\frac{1}{2}\bar{\psi}^i_{[\mu}\phi_{\nu]i} + \frac{3}{4}\bar{\psi}^i_{[\mu}\gamma_{\nu]}\chi_i + \text{h.c.}\right).
\end{aligned}
\tag{B.3}
$$

The following modified curvature tensors appear in the component fields of the chiral multiplet W^2 (cf. (2.7)),

$$
\begin{aligned}
\mathcal{R}(M)_{ab}{}^{cd} &= R(M)_{ab}{}^{cd} + \frac{1}{16}\left(T^{ijcd}\,T_{ijab} + T^{ij}_{ab}\,T^{cd}_{ij}\right), \\
\mathcal{R}(S)^i_{ab} &= R(S)^i_{ab} + \frac{3}{4}T^{ij}_{ab}\chi_j\,.
\end{aligned}
\tag{B.4}
$$

The T^2-modification cancels exactly the T^2-terms in the contribution to $\mathcal{R}(M)$ from $f_\mu{}^a$. The curvature $\mathcal{R}(M)_{ab}{}^{cd}$ satisfies the following relations,

$$
\begin{aligned}
\mathcal{R}(M)_{\mu\nu}{}^{ab}\,e^\nu{}_b &= i\tilde{R}(A)_\mu{}^a + \frac{3}{2}D\,e_\mu{}^a\,, \\
\frac{1}{4}\varepsilon_{ab}{}^{ef}\,\varepsilon^{cd}{}_{gh}\,\mathcal{R}(M)_{ef}{}^{gh} &= \mathcal{R}(M)_{ab}{}^{cd}\,, \\
\varepsilon_{cdea}\,\mathcal{R}(M)^{cd\,e}{}_b &= \varepsilon_{becd}\,\mathcal{R}(M)_a{}^{e\,cd} = 2\tilde{R}_{ab}(D) = 2iR_{ab}(A)\,.
\end{aligned}
\tag{B.5}
$$

The first one is the constraint that determines the field $f_\mu{}^a$ while the remaining equations are Bianchi identities. Note that the modified curvature does not satisfy the pair exchange property,

$$
\mathcal{R}(M)_{ab}{}^{cd} = \mathcal{R}(M)^{cd}{}_{ab} + 4i\delta^{[c}_{[a}\,\tilde{R}(A)_{b]}{}^{d]}\,. \tag{B.6}
$$

From these equations one determines for instance

$$
\mathcal{R}(M)^{\pm}_{0[p}{}^{0}{}_{q]} = \pm\frac{1}{2}iR(A)^{\pm}_{pq}\,. \tag{B.7}
$$

We note that $R(Q)^i_{ab}$ satisfies the constraint

$$
\gamma^\mu R(Q)^i_{\mu\nu} + \frac{3}{2}\gamma_\nu\chi^i = 0\,, \tag{B.8}
$$

which must therefore hold for its variation as well. This constraint implies that $R(Q)^i_{\mu\nu}$ is anti-selfdual, as follows from contracting it with $\gamma^\nu\gamma_{ab}$.

The curvature $\mathcal{R}(S)^i_{ab}$ satisfies

$$\gamma^a\tilde{\mathcal{R}}(S)^i_{ab} = 2\,D^a\tilde{R}(Q)^i_{ab}\,, \tag{B.9}$$

as a result of the Bianchi identities and of the constraint (B.8). This identity (upon contraction with $\gamma^b\gamma_{cd}$) leads to

$$\mathcal{R}(S)^i_{ab} - \tilde{\mathcal{R}}(S)^i_{ab} = 2\not{D}\left(R(Q)^i_{ab} + \frac{3}{4}\gamma_{ab}\chi^i\right). \tag{B.10}$$

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